Discussion
Large Sample Estimators of the Stochastic Discount Factor
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- This paper builds up on a recent working paper by Pukthuanthong and Roll (2016) (PR2016)

- Let $R^g_{i,t}$ denote the gross return on an asset i. As it is well known, standard asset pricing theory implies that there exisist a stochastic discount factor m_t such that

$$\mathrm{E}(m_t R^g_{i,t}) = 1$$

- PR2016 note that this implies that

$$\begin{aligned} \frac{1}{T}\sum_{t=1}^{T} m_t R^g_{i,t} &= \mathrm{E}(m_t R^g_{i,t}) + \frac{1}{T}\sum_{t=1}^{T}(m_t R^g_{i,t} - \mathrm{E}(m_t R^g_{i,t})) \\ &\to 1 \end{aligned}$$

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- This relation motivates PR2016 to introduce a new estimator of the series of the SDF m_t $t = 1, ..., T$
- Let $\mathbf{R}^g$ be the $N \times T$ matrix of gross asset returns and $\mathbf{m}$ the $T \times 1$ vector of observations of the SDF, then we have that

$$\frac{1}{T}\mathbf{R}^g\mathbf{m} = \mathbf{1}_N + \boldsymbol{\epsilon} \approx \mathbf{1}_N$$

- This implies that we can recover approximately the vector $\mathbf{m}$ by minimizing the pricing errors using

$$\mathbf{m}^{pr} = T(\mathbf{R}^{gT}\mathbf{R}^g)^{-1}\mathbf{R}^{gT}\mathbf{1}_N$$

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- PR approach
  - Pros: Approach is nonparametric as it imposes little structure on the problem
  - Cons: PR estimator may suffer from excessive variability

- This paper
  - provides estimators of the SDF **m** that aim at improving the estimation efficiency by assuming returns are generated by a factor model and using principal components

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- Let returns be generated by a factor model

$$\mathbf{R}^g = \mathbf{B}^g \mathbf{F}^g + \mathbf{E}^g$$

- Let $\mathbf{P}_k^g$ denote the matrix of the first k eigenvectors of the $\mathbf{R}^g \mathbf{R}^g / T$ matrix

- The Kim and Korajczyk (2017) estimator is

$$\mathbf{m}_K^{kk} = \mathbf{P}_k^g((\mathbf{R}^g \mathbf{P}_k^g / T)^T (\mathbf{R}^g \mathbf{P}_k^g / T))^{-1} (\mathbf{R}^g \mathbf{P}_k^g / T) \mathbf{1}_N$$

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Additional Results

- Paper analyses properties of this class of estimators and introduces different versions of the factor-based SDF estimator
- Carrys out at a realistic and extensive simulation study to assess the value added of the proposed methodology

Comment #1

- In a way, it is clear that the basic PR2016 can suffer from excessive variability and principal components is a natural approach to impose some regularization
- It maybe interesting to relate the proposed estimator to other shrinkage/regularization procedures in the literature
- In particular, a simple ridge type version of the PR estimator could be a more interesting benchmark

Comment #2

- It is not surprising to see that the proposed estimator performs better than the PR2016 benchmark in the simulation study
- It could be argued that the DGP used in the exercise (a strict k-factor model) is clearly realistic but is bound to produce results where the proposed estimator wins by a large margin
- Would it be possible to employ a DGP in which it is less clear that the proposed technology delivers large gains?

Comment #3

- Ultimately, it would be interesting to apply the proposed estimator to real data
- Do the authors have a strategy to assess the gains of more precise SDF estimation on real data?

Conclusions

Conclusion

- Interesting paper which I recommend reading
- Natural development of the PR2016 approach