

# Normal Distribution of Returns of 65 Stock Exchange Indexes

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# Outline

- 1 Summary
  - Remarks
- 2 Background
- 3 Questions

# Remarks

- **Straight-to-the-point and effective paper**

I enjoyed reading this paper.

- **Topic of interest:** Questions the well-known returns normality assumption in the literature.

Investigates in particular time frequency.

- **Many empirical results:** 65 equity indexes, 5 time intervals (daily, weekly, monthly, quarterly, yearly), 4 settings (CC, OO, OC, OV), 4 statistical tests (Jarque-Bera, Shapiro-Wilk, D'Agostino-Pearson). Performed 5,200 statistical tests.

# Outline

1 Summary

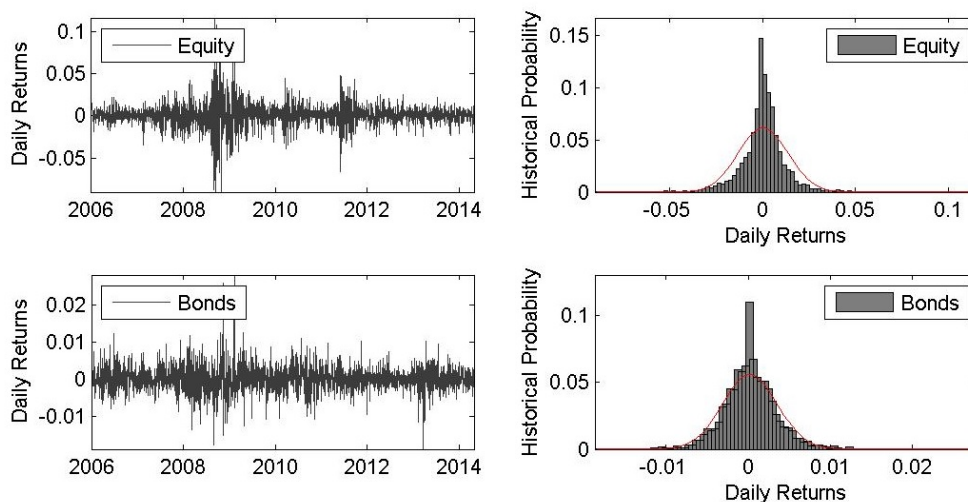
2 Background

- Motivation
- Contributions

3 Questions

# Motivation

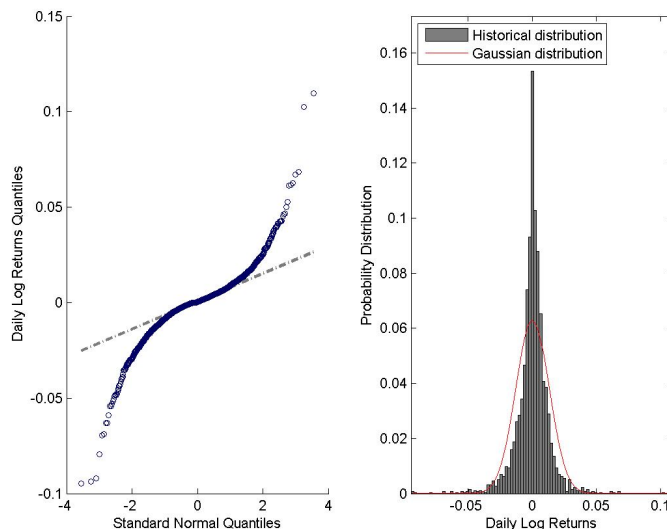
## Empirical Facts



**Figure 1:** Returns (left panels) and returns distribution (right panels) respectively associated to the S&P 500 total return index and the Barclays Global Government Bond Index over the time period 2006-2014. The histograms compare the empirical returns distribution to the Gaussian distribution (in orange). Empirical distributions depart from the normal one, exhibiting **fat tails**, **peakedness**, and **asymetry**.

# Motivation

## Empirical Facts



**Figure 2:** Returns (right panel) and returns Q-Q plot (left panel) associated to the S&P 500 total return index over the time period 2006-2014. The Q-Q plot compares empirical quantiles to standard normal quantiles, where straight line validates the normal assumption. Q-Q plot validates **non-normality**.

# Motivation

## Empirical Puzzle

**This paper addresses a major empirical puzzle:**

- **Returns non-normality is widely assumed:** Empirical returns distribution show **fat tails**, **peakedness**, and **asymetry**.
- **But high-frequency econometrics** comes up with a very different finding: this literature is recent and there are still many **grey areas** to explore.

**This paper provides additional and complementary empirical results documenting this empirical puzzle.**

# Contributions

- **Kendall (1953), Fama (1976), Barunik et al. (2010):** Clearly evidenced equity index daily returns are non-normally distributed.
- **Ghahfarokhi et al. (2009), Barunik et al. (2010), Piasecki et al. (2013):** Showed however that over specific time intervals / periods, normal distribution can be statistically assumed.

**Borowski (2017)** provides **new and broad evidence** over a wide range of **time intervals** (from daily to yearly), **time settings** (CC, OO, CO, OV), and **statistical tests** (JB, SW, AP). I particularly appreciated the smart **stock index ranking** procedure.

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- Robustness Checks
- Complementary Studies
- Implications

# Questions

## Robustness Checks

*(...) with a **strong increase in volatility** on 19.08.2015, the value of  $p$  dropped below the trigger value of 0.05. Explaining the decrease in the value of parameter  $p$  below 0.05 for  $K = 126$  and  $K = 252$  sessions becomes more complex issue and requires further investigation.*

## Economic interpretation behind: Extreme Events

You could control for **tail events** in your stock index ranking procedure. Intuitively, non-normality assumption should be widely rejected over extreme events, including Lehman Brothers bankruptcy, US sovereign debt crisis, EU Sovereign debt crisis, Flash Crash, Fed Tantrum, among others.

# Questions

## Complementary Studies

**Existing literature** about normality assumption is vast and have recourse to multiple complementary studies:

- Among others: Mandelbrot (1963), Danielsson et al. (2000), Thode (2002)

## Additional studies:

You could provide: *i*/ **summary statistics**, e.g. higher-order moments, parametric and historical VaR and CVaR; *ii*/ **graphical exploratory analysis**, e.g. histograms, Quantile-Quantile (Q-Q) plots, mean excess plots and kernel density estimation (KDE) plots.

# Questions

## Implications

What could be the **implications**?

- For risk management: tail-related measures, non-normal risk measures at stake (e.g. VaR, CVaR, Sharpe ratio);
- For asset pricing theory: e.g. CAPM, Markowitz;

Additional studies:

You could derive an **application** to risk management or asset pricing from your stock index ranking procedure.

*Thank you*