Realized Networks

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Introduction

High Frequency Data Based Volatility Estimation

- Over the last decade, the availability of intra-daily high frequency trade, quote and order book data has boosted research on the construction of efficient ex-post measure of daily return variability

- These estimator are typically called realized volatility estimators

- Extensive literature on the topic:
  Andersen, Bollerslev, Diebold and Labys (2003); Ait-Sahalia, Mykland and Zhang (2005); Bandi and Russell (2006); Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009);

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Two Challenges

- The multivariate generalizations of these estimators, aka realized covariance, have not not been as widely applied as their univariate counterparts

- Besides numerical challenges, realized covariance estimation suffers from two challenges which are inherently linked to covariance estimation for large number of assets:

  1. Precise estimation of the covariance (cf Ledoit & Wolf, 2004)
  2. Interpretation of the dependence structure of the assets

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In This Work...

1. We propose a LASSO–based regularization procedure for realized covariance estimation.
   - It shrinks the off diagonal elements of the inverse realized covariance to zero
   - Regularized estimator can be interpreted as a partial correlation network

   We call our estimator the Realized Network

2. We analyse the large sample properties of the estimator.
   - We employ standard covariance estimation framework which allows for market-microstructure noise and asynchronous trading
   - Focus on Two Scales Realized Covariance estimator and Multivariate Realized Kernel
   - Establish conditions of consistent covariance estimation and network selection

3. Advantages of the methodology are illustrated by means of a simulation study and an empirical illustration

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Related Literature

- Covariance Regularization

  Ledoit and Wolf (2004), Fan, Liao, Mincheva (2011), Ledoit and Wolf (2012), ...

  - Realized Covariance Regularization:

    Hautsch, Kyj and Oomen (2012); Corsi, Peluso, and Audrino (2015); Malec, Hautsch, Kyj (2015); Wang and Zhou (2010); Tao, Wang and Zhou (2013);

- Network Estimation in Econometrics and Statistics:

  Meinshausen & Buhlmann (2006); Brownlees & Barigozzi (2013); Diebold & Yilmaz (2013); Billio, Getmansky, Lo and Pellizzon (2012)

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Framework

Covariance of the Efficient Price

- $y(t)$ denotes the efficient log–price of n assets.
- $y(t)$ is a Brownian martingale

$$y(t) = \int_0^t \Theta(u) dB(u) \ ,$$

where $B(u)$ Brownian motion and $\Theta(u)$ is the spot covolatility

- Integrated Covariance: the covariance matrix of daily return $y(1)$

$$\text{Var}(y) = \int_0^1 \Sigma(t) dt = \Sigma^\star$$

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Partial Correlation Network

- We associate daily returns $y(1)$ with a partial correlation network
- The network associated with the system is an undirected graph

1. the components of $y(1)$ denote vertices
2. the presence of an edge between i and j denotes that i and j are [illegible] and the value of the partial correlation measures the strength of the link

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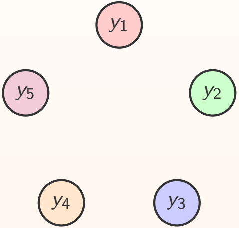

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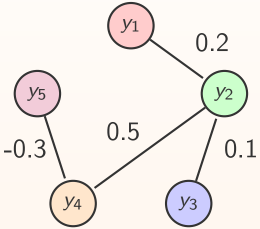

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Refresher on Partial Correlation

- Partial Correlation measures (cross-sect.) linear conditional dependence between $y_{i\,t}$ and $y_{j\,t}$ given on all other variables:

$$\rho^{ij} = \text{Cor}(y_{i\,t}, y_{j\,t} | \{y_{k\,t} : k \neq i, j\}).$$

- Partial Correlation is related to Linear Regression:
For instance, consider the model

$$y_{1\,t} = c + \beta_{1\,2} y_{2\,t} + \beta_{1\,3} y_{3\,t} + \beta_{1\,4} y_{4\,t} + \beta_{1\,5} y_{5\,t} + u_{1\,t}$$

β_{13} is different from 0 $\Leftrightarrow$ 1 and 3 are partially correlated

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If there is exist a partial correlation path between nodes i and j, then i and j are correlated (and viceversa).

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- Network is entirely characterized by the integrated concentration matrix $\mathbf{K}^\star = (\Sigma^*)^{-1} = (k^\star_{ij})$:

$$\rho^{ij} = \frac{-k^\star_{ij}}{\sqrt{k^\star_{ii} k^\star_{jj}}}$$

  In particular, the nonzero entries of $\mathbf{K}^*$ correspond to the linkages of the network.

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Factors

- In finance, it is customary to assume that returns have a factor structure. In practice, it is more interesting to analyse the partial correlation structure of assets conditional on the factors
- Thus, in this work we define the network on the basis of the idiosyncratic integrated covariance, that is the covariance of the residuals obtained by projecting the factors onto the assets

$$\Sigma_I^\star = \Sigma_{AA}^\star - \Sigma_{AF}^\star \left[\Sigma_{FF}^\star\right]^{-1} \Sigma_{FA}^\star = (\sigma_{I\,ij}^\star)$$

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Microstructure Noise & Asynchronous Trading

It is customary to assume that the econometrician does not observe the efficient price y but a contaminated version x defined as

$$x_i(t_{i\,k}) = y_i(t_{i\,k}) + u_i(t_{i\,k})$$

where

- $t_{i\,k}$ (asset specific) timestamp of a trade/midquote
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Estimation

- Objectives:
  1. Estimate the integrated covariance
  2. Detect the nonzero linkages of the network, which is equivalent to detecting the nonzero entries of the integrated concentration matrix.

- We are going to tackle both objectives simulatenously by introducing a sparse integrated concentration matrix estimator. More specifically:
  1. We are going to introduce an appropriate estimator of the integrated covariance
  2. and we are then going to regularize it by shrinking the off-diagonal entries of its inverse to zero via the LASSO.

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Two Scales Realized Covariance Estimator

- Our estimation strategy consist of regularizing a consistent RC estimator. Many estimators are available in the setting we are working on
- In this presentation we focus on the Two Scales Realized Covariance estimator $\bar{\Sigma}$ based on Pairwise–Refresh Time (TSRC). The estimator deals with both market microstructure noise and asynchronicity.
- Other sufficiently regular estimators can be used. Theory does not depend non specific form of the estimator. In particular, the paper also develops the theory for the Multivariate Realized Kernel.

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Two–Scale Realized Covariance (TSRC)

- Let $(x^r_{i\,k}, x^r_{j\,k})$ denote the "pairwise refresh time" adjusted observed prices for stock i and j
- The TSRC estimator is denoted by $\overline{\boldsymbol{\Sigma}}_{\text{TS}} = (\overline{\sigma}_{\text{TS},ij})$,

$$\overline{\sigma}_{\text{TS},ij} = \frac{1}{K}\sum_{k=K+1}^{m}\left(x^r_{i\,k} - x^r_{i\,k-K}\right)\left(x^r_{j\,k} - x^r_{j\,k-K}\right) - \frac{m_K}{m_J}\frac{1}{J}\sum_{k=J+1}^{m}\left(x^r_{i\,k} - x^r_{i\,k-J}\right)\left(x^r_{j\,k} - x^r_{j\,k-J}\right)$$

where $m_K = \frac{m-K+1}{K}$ and $m_J = \frac{m-J+1}{J}$.

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Realized Network Estimator

- The Realized Network Estimator is defined as

$$\widehat{\mathbf{K}} = \arg\min_{\mathbf{K}\in\mathcal{S}^n} \left\{ \mathrm{tr}(\overline{\Sigma}\mathbf{K}) - \log\det(\mathbf{K}) + \lambda \sum_{i\neq j} |k_{ij}| \right\}$$

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Theory

Preliminaries

Key ingredient to establish results is a concentration inequality of the realized volatility estimator.

- Define M as the minimum sample size used to compute a realized covariance entry across all pairs of stocks.

- Then we have

$$\mathrm{P}\left(\left|\overline{\sigma}_{ij} - \sigma^{\star}_{ij}\right| > x\right) \leq a_1 M^{\alpha} \exp\left\{-a_2 \left(M^{\beta} x\right)^{\gamma}\right\}.$$

for some positive exponents α, β, γ

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Consistent Estimation

Theorem: Consistent Concentration Estimation

Let $\lambda = \frac{8}{\alpha} M^{-\beta} \left(\frac{\log(a_1 n^{\tau})}{a_2} \right)^{\frac{1}{\gamma}}$ *for some* $\tau > 2$.

Let

$$M > C \left(\log \left(a_2 (a_1^{\frac{1}{\beta\gamma}} C_0(d)^{\frac{1}{\beta}}) n^{\tau} \right) \right)^{\frac{1}{\beta\gamma}} C_0(d)^{\frac{1}{\beta}},$$

where C_0 *is a function of the max vertex degree* d

Then, for n *sufficiently large*

$$\mathrm{P} \left(||\widehat{\mathbf{K}} - \mathbf{K}^{\star}||_{\infty} \leq 2 C_{\mathbf{\Gamma}^{*}} \left(1 + \frac{8}{\alpha} \right) M^{-\beta} \left[\frac{\log (a_1 M^{\alpha} n^{\tau})}{a_2} \right]^{\frac{1}{a_0}} \right) \geq 1 - \frac{1}{n^{\tau - 2}}$$

where $C_{\mathbf{\Gamma}^{\star}}$ *is a constant that depends on* Σ.

Consistent Selection

Theorem: Consistent Network Selection

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where C_1 *is a function of the max vertex degree* d.

Then, for n sufficiently large

$$\mathrm{P}\left(\mathrm{sign}(\widehat{k}_{ij}) = \mathrm{sign}(k^\star_{ij}), \forall i,j \in \{1,\dots,n\} \right) \geq 1 - \frac{1}{n^{\tau-2}}.$$

Estimator Precision and Sparsity

- It is useful to analyse how the expression simplify depending on the degree of sparsity of the networks

  1. If the max degree d is zero (disconnected graph), then the sample size M has to be at least $O((\log n)^{\frac{1}{\beta\gamma}})$
  2. If the max degree d is $O(n)$ (fully connected graph), then the sample size M has to be at least $O((\log n)^{\frac{1}{\beta\gamma}} n^{\frac{1}{\beta}})$

- Not that for the realized volatility estimator without noise and asynchronicity, $\beta = 1/2$

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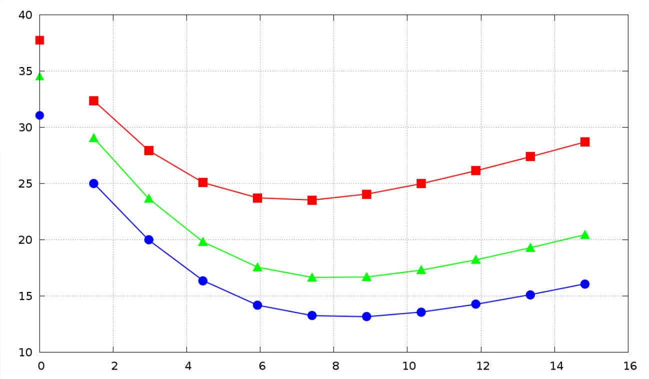
Simulation Study

Simulation Study Peek: MSE vs Network Shrinkage



- Simulation study to analyse finite sample properties of the procedure.
- In particular, study shows that if partial correlation structure is sparse, realized network estimator substantially enhances precision.

Empirical Application

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- We consider a panel of 96 NYSE Bluechips ($\approx$ constituents of the S&P 100)
- We estimate realized covariance for each day of 2009
- Realized covariance is estimated using the Realized Network estimator based on TSRC. (tuning parameter λ chosen via the BIC)
- Estimators are computed using trade prices from the NYSE–TAQ Standard procedures are applied to clean and filter the data
- We focus on the idiosyncratic covariance matrix We analyse interdependence conditional on the market factor.

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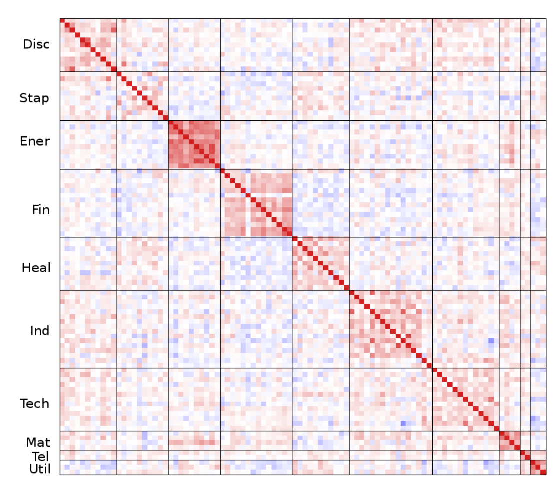
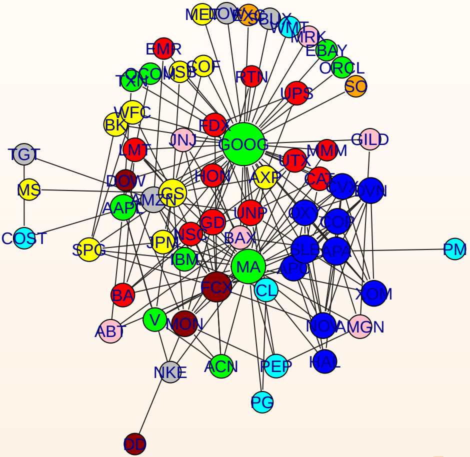
Realized Network Estimates

Realized Correlation Heatmap on 2009-07-02



Realized Network Estimates

Realized Network on 2009-07-02



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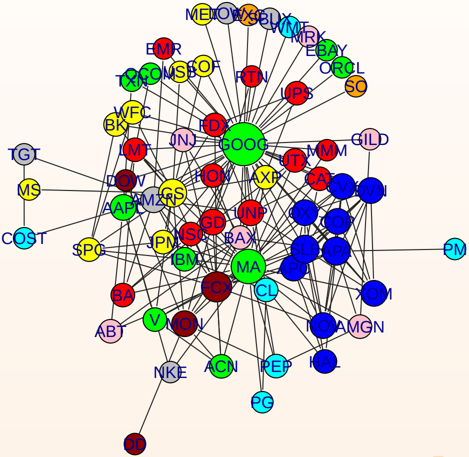
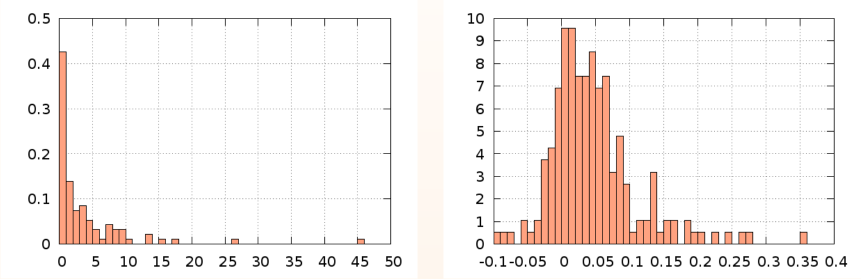


Degree and Partial Correlation Distribution



Degree

Partial Correlation

Predictive Analysis

- GMV portfolio prediction exercise:
  1. Construct the GMV portfolio weights using the MRK

     Competitors: Unconstrained, Constrained, Shrinkage and Realized Network
  2. Use the weights to construct daily GMV portfolio for the following day.
  3. Compute the variance of the daily portfolios over the full year

- More precise covariance estimators deliver GMV portfolio weights that generate smaller out-of–sample portfolio variances

  (cf Engle and Colacito, 2006)

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Predictive Analysis: GMV Comparison

	No Regular.	Diagonal	Network	Factor	Shrinkage	Block-Factor
RC	13.40	13.74	12.09	12.11	12.03	13.24
		-0.70	1.41	2.46^{**}	2.65^{***}	0.26
TSRC	13.05	13.52	11.96	12.34	12.16	12.29
		-0.74	1.88^{*}	1.34	1.73^{*}	0.88
MRK	12.95	13.89	11.65	13.03	10.46	11.15
		-1.82^{*}	2.17^{**}	-0.12	4.19^{***}	2.30^{**}

Conclusions

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- Propose a LASSO regularization procedure for realized covariance estimators. We call the regularized estimator Realized Network.
- Highlights:
  1. The procedure delivers more precise estimates of the covariance when the partial correlation structure of the assets is sparse.
  2. Regularized estimator can be represented as a network.
- Empirical application shows that regularization significantly improves the estimator. In a GMV portfolio forecasting exercise, substantial gains in prediction accuracy

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Questions?

Thanks!