The evaluation of combination of forecasts for realized volatility using asymmetric loss functions

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Introduction

- Volatility is a central parameter for many financial decisions including the pricing and hedging of derivative products as well as the development of efficient risk management methods.
- In literature there exists a wide variety of models that are able to estimate volatility forecasts, but they are, almost by definition, simple and incomplete (Raviv 2016).
- An improvement in the forecasts accuracy can be achieved combining forecasts originated from different types of models.

Introduction

The aim of this paper is:

- to forecast the daily realized volatility one-step-ahead for a one-year period with both single and combining models;
- to compare the predicted values with the actual data by means of a number of loss functions.

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Data

Three European market indexes:

1. DAX 30 (*Deutsche Aktienindex 30*);
2. CAC 40 (*Cotation Assistée en Continu*);
3. AEX (*Amsterdam Exchange Index*).

For each asset the realized volatility collected every 5 minutes, the realized kernel volatility and the daily returns are provided, covering the period from 01/01/2008 to 31/12/2016.

Methodology

Three different models have been chosen to create the single forecasts:

1. *Asymmetric Multiplicative Error Model* (AMEM);
2. *Asymmetric Power Multiplicative Error Model* (APMEM);
3. *Asymmetric Heterogeneous AutoRegressive Model* (AHAR).

Methodology

1. AMEM(1,1) model (Engle and Gallo 2006) has the following structure:

$$\begin{aligned} rv_t &= \mu_t \xi_t \\ \mu_t &= \omega + \alpha rv_{t-1} + \beta \mu_{t-1} + \gamma D_{t-1} rv_{t-1} \end{aligned}$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\gamma \geq 0$, $\alpha + \beta + \frac{\gamma}{2} < 1$.

Methodology

2. APMEM(1,1) model is given by:

$$
\begin{aligned}
rv_t &= \mu_t \xi_t \\
\mu_t^\delta &= \omega + \alpha rv_{t-1}^\delta + \beta \mu_{t-1}^\delta + \gamma D_{t-1} rv_{t-1}^\delta
\end{aligned}
$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$, $\delta > 0$.

Methodology

3. AHAR is the HAR model (Corsi 2009) with a leverage effect term:

$$rv_t = c + \beta^{(d)} rv_{t-1} + \beta^{(w)} rv_{t-1}^{(w)} + \beta^{(m)} rv_{t-1}^{(m)} + \epsilon_t^{(d)}$$

where:

- (d) stands for the time horizons of one day;
- $rv_{t-1}^{(w)}$ is the weekly realized volatility which at time t is given by the average

$$rv_t^{(w)} = \frac{1}{5}\left(rv_t^{(d)} + rv_{t-1d}^{(d)} + \cdots + rv_{t-4d}^{(d)}\right)$$

- $rv_{t-1}^{(m)}$ is the monthly realized volatility which at time t is given by the average

$$rv_t^{(m)} = \frac{1}{22}\left(rv_t^{(d)} + rv_{t-1d}^{(d)} + \cdots + rv_{t-21d}^{(d)}\right)$$

Figure: Comparison among observed realized volatility (5 minutes) for year 2016 and AMEM(1,1), APMEM(1,1) and AHAR forecasts - DAX dataset

Methodology

The combining methods are based on the following two combination models:

- *comb1* model, based on a simple unconstrained Ordinary Least Squares estimates of the weights. The one-step-ahead forecast is given by

$$rv_T(1) = \alpha + \beta_1 f_T^{(1)}(1) + \beta_2 f_T^{(2)}(1)$$

with $f_T^{(1)}(1)$ and $f_T^{(2)}(1)$ denote, respectively, the first and second model forecasts.

Methodology

- *comb2* model, with the combination given by

$$rv_T(1) = \alpha + (\beta_1 + \delta_1 D_{t-1}) f_T^{(1)}(1) + (\beta_2 + \delta_2 D_{t-1}) f_T^{(2)}(1)$$

which includes a dummy variable D_t:

$$D_t = \begin{cases} 1 & \text{if } rv_t < rv_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

Loss Functions

To compare the results of the combination schemes with those that can be reached by exclusively relying on a single model, we compute five loss functions:

1. *Mean Square Error* (MSE);
2. *Mean Absolute Error* (MAE);

Loss Functions

3. *Quasi-Likelihood* (QLIKE), defined as

$$\frac{1}{n}\sum_{i=1}^{n}\left[\frac{rv_{T+i}}{rv_{T+i-1}(1)} - \ln\left(\frac{rv_{T+i}}{rv_{T+i-1}(1)}\right) - 1\right]$$

with rv_{T+i} being the observed value of the realized volatility and $rv_{T+i-1}(1)$ is the one-step-ahead forecast for time $T+i$, $i = 1, \ldots, n$.

Loss Functions

4. a first new measure called *Asymmetric Mean Square Error* (AMSE), given by

$$\frac{1}{n}\sum_{i=1}^{n}\left(1+\left(\frac{\epsilon_{T+i}{}^{2}}{rv_{T+i}}\right)^{m}\mathcal{I}(\epsilon_{T+i}>0)\right)\epsilon_{T+i}{}^{2}$$

where $\epsilon_{T+i} = rv_{T+i} - rv_{T+i-1}(1)$.

Loss Functions

5. a second original measure called *Asymmetric Mean Absolute Error* (AMAE), given by

$$\frac{1}{n}\sum_{i=1}^{n}\left(1+\left(\frac{|\epsilon_{T+i}|}{rv_{T+i}}\right)^{m}\mathcal{I}(\epsilon_{T+i}>0)\right)|\epsilon_{T+i}|$$

where, as before, $\epsilon_{T+i} = rv_{T+i} - rv_{T+i-1}(1)$.

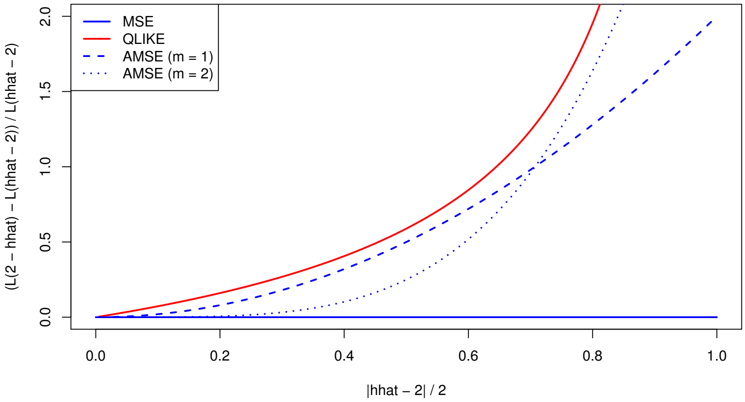

Figure: Comparison among MSE, QLIKE, AMSE (m=1,2) loss functions computed on a series *h* of evenly spaced forecasts from 0 to 2.

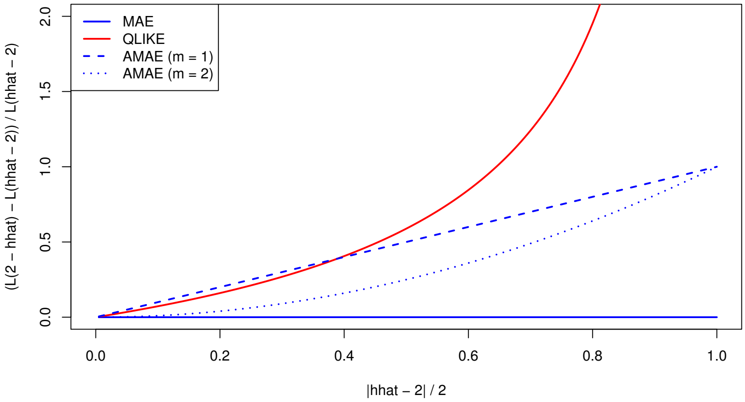

Figure: Comparison among MAE, QLIKE, AMAE (m=1,2) loss functions computed on a series *h* of evenly spaced forecasts from 0 to 2.

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		MSE				MAE				QLIKE			
Series	Training period	AMEM (1,1)	AHAR	comb1	comb2	AMEM (1,1)	AHAR	comb1	comb2	AMEM (1,1)	AHAR	comb1	comb2
rv 5min	4 years	0.254	0.293	0.254	**0.248**	3.459	3.671	3.458	**3.361**	**4.231**	4.979	4.235	4.246
	3 years	0.254	0.293	0.254	**0.250**	3.459	3.671	3.461	**3.418**	**4.231**	4.979	4.244	4.338
rv kernel	4 years	0.206	0.251	0.206	**0.200**	3.124	3.386	3.125	**3.021**	3.505	4.312	**3.505**	3.508
	3 years	0.206	0.251	0.206	**0.203**	3.124	3.386	3.126	**3.082**	**3.505**	4.312	3.511	3.594
		AMSE ($m = 1$)				AMAE ($m = 1$)							
Series	Training period	AMEM (1,1)	AHAR	comb1	comb2	AMEM (1,1)	AHAR	comb1	comb2				
rv 5min	4 years	0.271	0.316	0.272	**0.268**	3.975	4.239	3.975	**3.903**				
	3 years	0.271	0.316	0.272	**0.270**	3.975	4.239	3.978	**3.939**				
rv kernel	4 years	0.217	0.267	0.217	**0.212**	3.556	3.886	3.557	**3.476**				
	3 years	0.217	0.267	0.218	**0.215**	3.556	3.886	3.559	**3.521**				
		AMSE ($m = 2$)				AMAE ($m = 2$)							
Series	Training period	AMEM (1,1)	AHAR	comb1	comb2	AMEM (1,1)	AHAR	comb1	comb2				
rv 5min	4 years	0.257	0.298	0.257	**0.251**	3.679	3.925	3.679	**3.596**				
	3 years	0.257	0.298	0.258	**0.254**	3.679	3.925	3.682	**3.644**				
rv kernel	4 years	0.207	0.254	0.208	**0.201**	3.296	3.599	3.297	**3.205**				
	3 years	0.207	0.254	0.208	**0.204**	3.296	3.599	3.299	**3.259**				

Table: Comparison among AMEM(1,1), AHAR and combination schemes (in bold the smallest values) - DAX dataset

Series	Training period	MSE AMEM (1,2)	MSE AHAR	MSE comb1	MSE comb2	MAE AMEM (1,2)	MAE AHAR	MAE comb1	MAE comb2	QLIKE AMEM (1,2)	QLIKE AHAR	QLIKE comb1	QLIKE comb2
rv 5min	4 years	0.243	0.292	0.243	**0.239**	3.164	3.502	3.169	**3.098**	**3.809**	4.660	3.831	3.836
	3 years	0.243	0.292	0.243	**0.242**	3.164	3.502	3.169	**3.126**	**3.809**	4.660	3.838	3.891
rv kernel	4 years	0.240	0.295	0.240	**0.233**	3.150	3.546	3.161	**3.045**	3.855	4.810	3.877	**3.834**
	3 years	0.240	0.295	0.241	**0.236**	3.150	3.546	3.160	**3.083**	**3.855**	4.810	3.878	3.895

Series	Training period	AMSE ($m = 1$) AMEM (1,2)	AHAR	comb1	comb2	AMAE ($m = 1$) AMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.268	0.325	0.269	**0.267**	3.632	4.046	3.638	**3.593**
	3 years	**0.268**	0.325	0.270	0.270	3.632	4.046	3.639	**3.616**
rv kernel	4 years	0.264	0.325	0.265	**0.260**	3.619	4.096	3.631	**3.534**
	3 years	0.264	0.325	0.265	**0.263**	3.619	4.096	3.631	**3.567**

Series	Training period	AMSE ($m = 2$) AMEM (1,2)	AHAR	comb1	comb2	AMAE ($m = 2$) AMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.249	0.301	0.249	**0.246**	3.375	3.752	3.381	**3.323**
	3 years	0.249	0.301	0.250	**0.249**	3.375	3.752	3.383	**3.349**
rv kernel	4 years	0.245	0.303	0.246	**0.240**	3.364	3.800	3.376	**3.270**
	3 years	0.245	0.303	0.246	**0.242**	3.364	3.800	3.375	**3.306**

Table: Comparison among AMEM(1,2), AHAR and combination schemes (in bold the smallest values) - CAC dataset

Series	Training period	MSE AMEM (1,2)	MSE AHAR	MSE comb1	MSE comb2	MAE AMEM (1,2)	MAE AHAR	MAE comb1	MAE comb2	QLIKE AMEM (1,2)	QLIKE AHAR	QLIKE comb1	QLIKE comb2
rv 5min	4 years	0.251	0.293	0.251	**0.247**	3.003	3.170	2.993	**2.960**	**3.925**	4.706	3.941	3.960
	3 years	**0.251**	0.293	0.251	0.251	3.003	3.170	**2.995**	3.006	**3.925**	4.706	3.949	4.023
rv kernel	4 years	0.219	0.257	0.219	**0.213**	2.947	3.180	2.948	**2.884**	3.854	4.677	3.872	**3.834**
	3 years	0.219	0.257	0.219	**0.216**	2.947	3.180	2.950	**2.922**	**3.854**	4.677	3.874	3.886

Series	Training period	AMSE ($m = 1$) AMEM (1,2)	AHAR	comb1	comb2	AMAE ($m = 1$) AMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.288	0.343	0.289	**0.287**	3.488	3.699	3.477	**3.454**
	3 years	**0.288**	0.343	0.290	0.291	3.488	3.699	**3.479**	3.493
rv kernel	4 years	0.241	0.286	0.241	**0.237**	3.404	3.691	3.405	**3.348**
	3 years	0.241	0.286	0.242	**0.239**	3.404	3.691	3.407	**3.378**

Series	Training period	AMSE ($m = 2$) AMEM (1,2)	AHAR	comb1	comb2	AMAE ($m = 2$) AMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.263	0.313	0.263	**0.261**	3.233	3.434	3.225	**3.198**
	3 years	**0.263**	0.313	0.264	0.265	3.233	3.434	**3.228**	3.240
rv kernel	4 years	0.224	0.266	0.224	**0.218**	3.153	3.419	3.155	**3.095**
	3 years	0.224	0.266	0.225	**0.221**	3.153	3.419	3.157	**3.129**

Table: Comparison among AMEM(1,2), AHAR and combination schemes (in bold the smallest values) - AEX dataset

		MSE				MAE				QLIKE			
Series	Training period	APMEM (1,1)	AHAR	comb1	comb2	APMEM (1,1)	AHAR	comb1	comb2	APMEM (1,1)	AHAR	comb1	comb2
rv 5min	4 years	0.249	0.293	0.249	**0.245**	3.421	3.671	3.418	**3.332**	4.165	4.979	**4.152**	4.156
	3 years	0.249	0.293	0.249	**0.247**	3.421	3.671	3.421	**3.383**	4.165	4.979	**4.162**	4.232
rv kernel	4 years	0.203	0.251	0.203	**0.198**	3.101	3.386	3.099	**3.006**	3.464	4.312	3.448	**3.429**
	3 years	0.203	0.251	0.203	**0.200**	3.101	3.386	3.101	**3.059**	3.464	4.312	**3.456**	3.506

		AMSE ($m = 1$)				AMAE ($m = 1$)			
Series	Training period	APMEM (1,1)	AHAR	comb1	comb2	APMEM (1,1)	AHAR	comb1	comb2
rv 5min	4 years	0.267	0.316	0.266	**0.265**	3.933	4.239	3.931	**3.867**
	3 years	0.267	0.316	0.267	**0.266**	3.933	4.239	3.934	**3.901**
rv kernel	4 years	0.214	0.267	0.214	**0.210**	3.534	3.886	3.532	**3.457**
	3 years	0.214	0.267	0.214	**0.213**	3.534	3.886	3.533	**3.499**

		AMSE ($m = 2$)				AMAE ($m = 2$)			
Series	Training period	APMEM (1,1)	AHAR	comb1	comb2	APMEM (1,1)	AHAR	comb1	comb2
rv 5min	4 years	0.252	0.298	0.252	**0.249**	3.640	3.925	3.637	**3.564**
	3 years	0.252	0.298	0.252	**0.251**	3.640	3.925	3.640	**3.607**
rv kernel	4 years	0.205	0.254	0.204	**0.200**	3.274	3.599	3.271	**3.187**
	3 years	0.205	0.254	0.205	**0.202**	3.274	3.599	3.273	**3.236**

Table: Comparison among APMEM(1,1), AHAR and combination schemes (in bold the smallest values) - DAX dataset

		MSE				MAE				QLIKE			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.246	0.292	0.245	**0.240**	3.177	3.502	3.191	**3.091**	**3.833**	4.660	3.868	3.855
	3 years	0.246	0.292	0.246	**0.242**	3.177	3.502	3.193	**3.113**	**3.833**	4.660	3.877	3.913
rv kernel	4 years	0.241	0.295	0.242	**0.234**	3.158	3.546	3.172	**3.045**	3.865	4.810	3.893	**3.840**
	3 years	0.241	0.295	0.242	**0.236**	3.158	3.546	3.173	**3.077**	**3.865**	4.810	3.899	3.903

		AMSE ($m = 1$)				AMAE ($m = 1$)			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.272	0.325	0.272	**0.268**	3.646	4.046	3.662	**3.588**
	3 years	0.272	0.325	0.273	**0.271**	3.646	4.046	3.665	**3.607**
rv kernel	4 years	0.266	0.325	0.266	**0.261**	3.628	4.096	3.644	**3.536**
	3 years	0.266	0.325	0.267	**0.263**	3.628	4.096	3.646	**3.563**

		AMSE ($m = 2$)				AMAE ($m = 2$)			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.252	0.301	0.252	**0.247**	3.389	3.752	3.405	**3.318**
	3 years	0.252	0.301	0.253	**0.249**	3.389	3.752	3.407	**3.340**
rv kernel	4 years	0.247	0.303	0.247	**0.240**	3.372	3.800	3.387	**3.271**
	3 years	0.247	0.303	0.248	**0.243**	3.372	3.800	3.389	**3.301**

Table: Comparison among APMEM(1,2), AHAR and combination schemes (in bold the smallest values) - CAC dataset

		MSE				MAE				QLIKE			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.256	0.293	0.254	**0.247**	3.022	3.170	3.003	**2.945**	**3.946**	4.706	3.971	3.975
	3 years	0.256	0.293	0.254	**0.250**	3.022	3.170	3.005	**2.989**	**3.946**	4.706	3.980	4.039
rv kernel	4 years	0.219	0.257	0.219	**0.213**	2.950	3.180	2.949	**2.878**	3.855	4.677	3.875	**3.831**
	3 years	0.219	0.257	0.220	**0.215**	2.950	3.180	2.950	**2.916**	**3.855**	4.677	3.879	3.886

		AMSE ($m=1$)				AMAE ($m=1$)			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.292	0.343	0.293	**0.288**	3.508	3.699	3.487	**3.443**
	3 years	0.292	0.343	0.294	**0.292**	3.508	3.699	3.490	**3.479**
rv kernel	4 years	0.241	0.286	0.242	**0.237**	3.407	3.691	3.407	**3.345**
	3 years	0.241	0.286	0.242	**0.239**	3.407	3.691	3.408	**3.375**

		AMSE ($m=2$)				AMAE ($m=2$)			
Series	Training period	APMEM (1,2)	AHAR	comb1	comb2	APMEM (1,2)	AHAR	comb1	comb2
rv 5min	4 years	0.267	0.313	0.267	**0.261**	3.253	3.434	3.236	**3.186**
	3 years	0.267	0.313	0.267	**0.264**	3.253	3.434	3.239	**3.226**
rv kernel	4 years	0.224	0.266	0.224	**0.219**	3.156	3.419	3.156	**3.091**
	3 years	0.224	0.266	0.225	**0.221**	3.156	3.419	3.158	**3.125**

Table: Comparison among APMEM(1,2), AHAR and combination schemes (in bold the smallest values) - AEX dataset

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Conclusions

- We have found that combining the AHAR model with APMEM instead of AMEM causes an improvement in the accuracy of the forecasts computed using combination schemes, especially the *comb2* model.
- This finding holds for DAX and AEX datasets and for all training periods, whereas for the CAC index there was not any change in loss function choises when moving from AMEM to APMEM.

References

- Corsi, Fulvio (2009). "A simple approximate long-memory model of realized volatility". In: *Journal of Financial Econometrics* 7.2, pp. 174–196.
- Ding, Zhuanxin, Clive WJ Granger, and Robert F Engle (1993). "A long memory property of stock market returns and a new model". In: *Journal of empirical finance* 1.1, pp. 83–106.
- Engle, Robert F and Giampiero M Gallo (2006). "A multiple indicators model for volatility using intra-daily data". In: *Journal of Econometrics* 131.1, pp. 3–27.
- Raviv, E. (2016). "Forecast combinations in R using the ForecastCombinations package A Manual".

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