NORMAL DISTRIBUTION OF RETURNS OF 65 STOCK EXCHANGE INDEXES

dr hab. prof. SGH Krzysztof Borowski

Assumption of normal distribution of rates of return on financial markets

For example

- Portfolio theory (Markowitz),
- CAPM
- Black-Scholes model of valuating options

The aim of the paper

To verify the hypothesis on the normal distribution of:

a) daily,

b) weekly,

c) monthly,

d) quarterly

e) yearly

of 65 stock index returns.

Literature

- Bachelier (1900)
- Kendall (1953)
- Osborne (1959)
- Scalas, Kim (2007)
- Barunik, Vacha, Vosvrd (2010)
- Ghahfarokhi i Ghahfarokhi (2009)

Literature

- On the Polish market
  - Bołt, Miłobędzki (1994),
  - Fiszeder (2000),
  - Rokita (2000),
  - Osińska (2006),
  - Witkowska, Kompa (2007)

Methodology

- The paper consists of 3 parts
- Rates of return:

For each of the analyzed indices the following rates of return were calculated (daily rates of return):

a) Close – Close (C-C): $ln\left(\frac{C_t}{C_{t-1}}\right)$ (last session close vs previous session close)

b) Overnight (OV): $ln\left(\frac{O_t}{C_{t-1}}\right)$ (last session open vs previous session close)

c) Open – Open (O-O): $ln\left(\frac{O_t}{O_{t-1}}\right)$ (last session open vs previous session open)

d) Open – Close (O-C): $ln\left(\frac{C_t}{O_t}\right)$ (last session close vs last session open)

where:

C_t – closing price in the period t,

C_{t-1} – closing price in the period $t-1$,

O_t – open price in the period t,

O_{t-1} – open price in the period $t-1$.

Methodology 2

The choice of the above rates of return results from two premises:

- a transaction takes place at strictly defined moments of the session at the opening or closing prices.
- derives of earlier scientific papers, because most research concentrates solely on the close-close rates.

Methodology – time horizon

a) Since the first session to 31.06.2017
b) For the following rates of return:
    a) daily,
    b) weekly,
    c) monthly,
    d) quarterly
    e) yearly

Methodology

- Test:
  - Jarque-Bera (JB),
  - Lilliefors (L),
  - Cramer von Mises (CVM),
  - Watson (W)
  - Anderson-Darling (AD).
- *p* (*p value*) calculated

The hypothesis

- The hypothesis H_0 was formulated as follows: the distribution of the analyzed index returns is a normal distribution.
- The alternative hypothesis H_1 takes the following form: the distribution of the analyzed index returns does not follow a path of a normal distribution.

Methodology – part II of research

- The hypothesis of the normal distribution of return rates for the following indices: CAC40, DAX, DJIA, FTSE250, NIKKEI and S&P 500 in the period 2013-2016 was verified (for each year)
- For DJIA selected 28 up and down waves

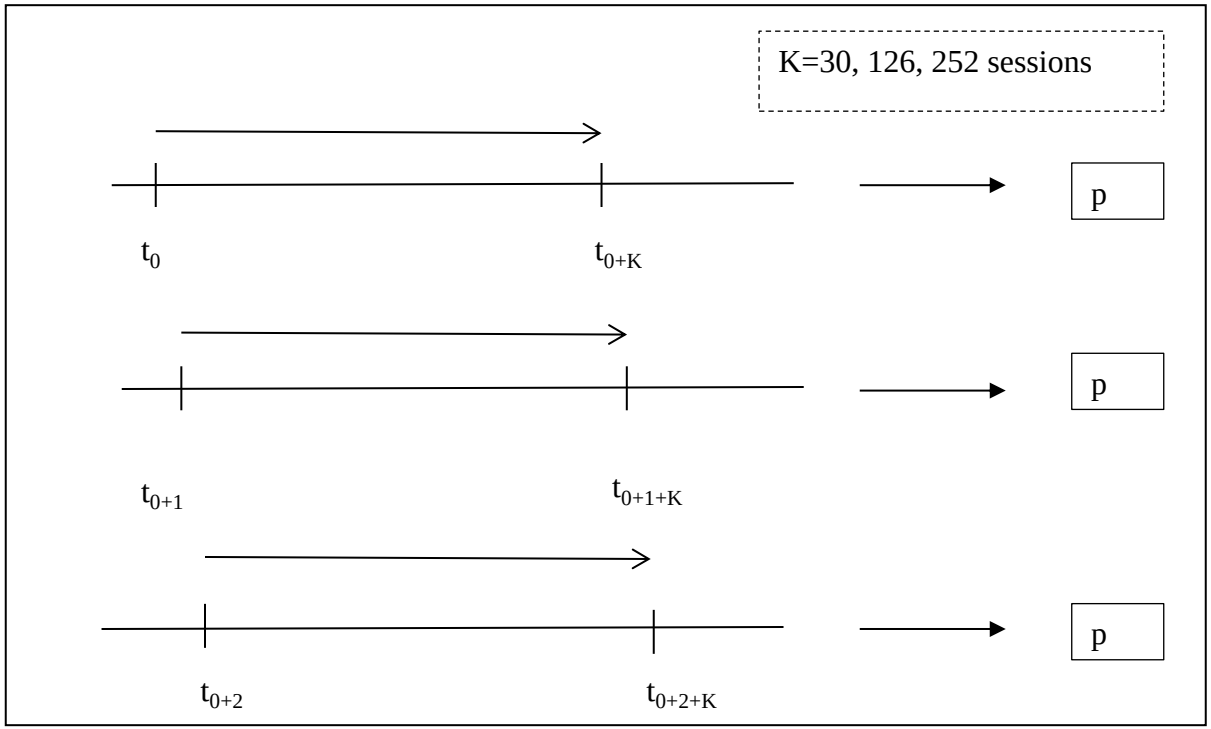
Methodology – part II

- The implemented test in the second part:
  - Jarque-Bera,
  - Kołmorgow-Smirnow,
  - Lilliefors,
  - Cramer von Mises,
  - Watson,
  - Anderson-Darling

Methodology – part II

- The purpose of this study is to demonstrate that the distribution of rates of return for individual indices can be normal in shorter time intervals.
- Part two of the study can be considered as an introduction to the third part.

Metodologia – part III



- *p* – as a metrics
- *p* – calculated for tests: Jarque-Bera, Shapiro-Wilk and D'Agostino-Pearson.

Metodologia – part III

- Frequency of p>0,05 for each of the tests, for each K and for each of returns
- (together 3x3x4=36 statistics)

Methodology– rankings

$$S_{I+II+II} = S_I + S_{II} + S_{III}$$

where:

S_I - position in the ranking of a given index for Jarque-Bera test,

S_{II} - position in the ranking of a given index for Shapiro-Wilk test,

S_{III} - position in the ranking of a given index for D'Agostino-Pearson test,

As a result 12 rankings were obtained: (4 rates of return: C-C, O-O, O-C and overnight) x (3 investment horizons K: 30, 126 and 252 sessions). Then on the basis of these 12 ratings, the following sum of the ratings was calculated for each of analyzed indexes:

Global ranking →

$$S_{I+\cdots XII} = {}^{K=30}_{C-C}S + {}^{K=30}_{O-O}S + {}^{K=30}_{O-C}S + {}^{K=30}_{OV}S + {}^{K=100}_{C-C}S + {}^{K=100}_{O-O}S + {}^{K=100}_{O-C}S + {}^{K=100}_{OV}S +$$
$$+ {}^{K=252}_{C-C}S + {}^{K=252}_{O-O}S + {}^{K=252}_{O-C}S + {}^{K=252}_{OV}S$$

where:

${}^{K}_{X}S$ - position of a given index in the ranking for a specific time horizon K (K = 30, 126 and 252 sessions) and return type X (C-C, O-O, O-C and overnight).

The sums $S_{I+\cdots XII}$ for each of analyzed indexes were used in the process of a global ranking construction.

Main and auxiliary thesis

- The main thesis of the analysis has been formulated as follows: in long time intervals, the returns distributions are not normal distributions.
- As a long time interval, investment horizon covering several years was assumed.
- In turn, the secondary thesis of the research may be expressed as follow: in the shorter investment horizons, the distribution of equity indexes returns may be normal.
- The auxiliary thesis can also be written in a slightly different way: returns of equity indexes are serially normal.

Results

Results – I part

- In case of the daily and weekly rates or return the hypothesis H_0 was rejected in favor of the hypothesis H_1 (for all indexes).
- In case of monthly rates of return there was no reason to reject the H_0 hypothesis for 42 indexes but only for:
  - 4 indexes (C-C)
  - 2 indexes (O-O)
  - 3 indexes (O-C)
- the result obtained with the use of one test were confirmed by results given by another statistical test – see table 1.

Table 1. The value of the parameter *p* for indices for which the obtained results with the use of one test were confirmed by the second test. Values of p> 0.05 marked in bold.

	C-C					O-O					O-C				
Index	**J-B**	**L**	**CVM**	**W**	**A-D**	**J-B**	**L**	**CVM**	**W**	**A-D**	**J-B**	**L**	**CVM**	**W**	**A-D**
FTSEMIB	0.0415	**0.1**	0.0164	**0.165**	0.0074	0.024	0.0107	0.0018	0.0016	0.0014	**0.1016**	**0.1**	0.0275	0.0282	0.0307
PSI20	0	**0.1**	**0.0784**	**0.0907**	0.039	0	**0.0755**	0.0362	0.0334	0.0072	0	0	0	0	0
SESESLCT	**0.4831**	**0.1**	**0.683**	**0.6672**	**0.6437**	**0.4757**	**0.1**	**0.7173**	**0.6995**	**0.6592**	**0.4799**	**0.1**	**0.6998**	**0.6848**	**0.655**
TOPIX	0	**0.0767**	**0.0775**	**0.118**	**0.1004**	0.0005	**0.1**	**0.2413**	**0.3324**	**0.3128**	0	**0.1**	**0.07**	**0.1204**	**0.0837**

Source: own calculations

Quarterly rates of return

- For quarterly rates of return the number of cases when there was no reason to reject the H_0 hypothesis was as follows:

- C-C (16): BUX, FTSE MIB, IBEX35, IPC, MEXICIPC, NZX50, PSI20, RUSSEL*, SAX, SDAX, SESESLCT, SSEBSHARES, TAIEX*, TOPIX, UX, XU100,
- O-O (19): BUX, FTSE MIB, IBEX35, IPC, MEXIXIPC, NZX50, OMXTALIN, PSI20, PSEI20*, RUSSEL, SAX, SDAX, SENSEX*, SESESLCT, SSEBSHARE, TAIEX, TOPIX, UX*, XU100,
- O-C (17): BUX, FTSE MIB, IBEX35, IPC, MEXIXIPC, NZX50, PSI20, RUSSEL*, SAX, SDAX, SESESLCT, SET*, SSEBSHARE, TAIEX, TOPIX, UX, XU100,
- Overnight (4): EOE, HEX, TEXCADX, TOPIX.
- With "*" are marked these indexes when the rejection of the H_0 hypothesis was obtained with the use of one test only.

Yearly rates of return

- For yearly rates of return the number of cases when there was no reason to reject the H_0 hypothesis was equal to: 55 (8), 51 (3), 52 (6), 21 (6) for C-C, O-O, O-C and overnight rates of return, respectively.
- The number of cases in parentheses is given when the null hypothesis was rejected by only one test.

Yearly rates of return – part 1

Table 3. The value of the parameter *p* for yearly equity index rates of return. Values of p> 0.05 marked in bold.

Indeks	Data	C-C					O-O					O-C					Overnight				
		J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D
AEX	03.01.1983	0.003	0.0114	0.0221	0.0349	0.021	0.0049	0.0111	0.021	0.0341	0.0231	0.0034	0.0092	0.0242	0.0372	0.0252	0.0369	0.1	0.2814	0.2699	0.3754
ALL Ordinaries	01.01.1900	0	0.0012	0.0001	0	0.0001	0	0.0031	0.0001	0.0001	0.0001	0	0.0012	0.0001	0	0.0001	0	0	0	0	0
Athex Com	02.01.1987	0.5136	0.1	0.6804	0.7243	0.7225	0.5499	0.1	0.5993	0.6308	0.6983	0.5091	0.1	0.6107	0.6546	0.6833	0.0021	0.0004	0	0	0
BEL20	02.01.1991	0	0.1	0.043	0.0718	0.0194	0	0.0952	0.0659	0.1096	0.0338	0	0.0532	0.0405	0.0691	0.0178	0.0097	0.0217	0.0339	0.0551	0.033
BET	31.10.2000	0.0003	0.0134	0.0175	0.0176	0.0115	0.0002	0.0266	0.0131	0.0129	0.009	0.0001	0.0084	0.0084	0.0079	0.0061	0	0	0	0	0
Bovespa	12.07.1989	0.0001	0	0	0	0	0	0.0002	0	0	0	0	0	0	0	0	0	0	0	0	0
Bux	02.01.1991	0.404	0.1	0.3827	0.3448	0.3352	0.4257	0.1	0.4228	0.3848	0.3941	0.4126	0.1	0.4453	0.4034	0.3997	0.0005	0.0001	0	0	0
CAC40	08.01.1965	0.3466	0.1	0.2276	0.2694	0.2524	0.579	0.1	0.5551	0.5839	0.6387	0.454	0.1	0.3151	0.3527	0.3866	0	0	0	0	0
CDAX	15.03.2004	0.0088	0.0514	0.014	0.0196	0.0081	0.0058	0.0446	0.0034	0.0043	0.0022	0.009	0.0368	0.0137	0.019	0.0081	0.8119	0.1	0.8959	0.8871	0.8716
DAX	28.09.1959	0.2916	0.0908	0.1746	0.2253	0.2154	0.1802	0.033	0.0899	0.1267	0.1169	0.2226	0.0941	0.1709	0.2179	0.2109	0	0.0355	0.0071	0.0151	0.0031
DJCA	23.12.1980	0.0146	0.075	0.0441	0.0764	0.0394	0.026	0.067	0.0354	0.0575	0.0389	0.0186	0.0827	0.0546	0.094	0.0505	0	0	0	0	0
DJIA	02.01.1900	0.0002	0.0288	0.0066	0.0107	0.0051	0.0003	0.0552	0.0131	0.0226	0.0098	0.0002	0.0244	0.0074	0.0125	0.0053	0	0	0	0	0
DJTA	02.01.1929	0	0.1	0.1633	0.2622	0.079	0	0.1	0.1363	0.2349	0.0662	0	0.1	0.1534	0.2476	0.0744	0	0	0	0	0
DJUA	02.01.1929	0.0007	0.0096	0.003	0.0041	0.0034	0.0029	0.0094	0.0032	0.0045	0.0037	0.0008	0.0091	0.0038	0.0051	0.0041	0	0	0	0	0
EOE	02.01.1995	0.0006	0.0004	0.0024	0.0031	0.0026	0.0024	0.002	0.012	0.0183	0.0102	0.0007	0.0004	0.0029	0.0038	0.0033	0.9788	0.1	0.8481	0.8213	0.915
FTSE100	22.10.1992	0.0969	0.0784	0.0181	0.0258	0.0174	0.1273	0.1	0.0432	0.0637	0.0402	0.1053	0.0759	0.0227	0.0324	0.0225	0	0	0	0	0
FTSE250	31.12.1985	0.0139	0.1	..1429	0.2277	0.097	0.0117	0.1	0.0739	0.1317	0.0479	0.017	0.0952	0.1547	0.2424	0.1029	0	0	0	0	0
FTSEMIB	02.01.1999	0.0428	0.0832	0.0196	0.0261	0.0119	0.0284	0.0231	0.0093	0.0117	0.006	0.0262	0.1	0.0161	0.0218	0.0099	0.4554	0.1	0.0713	0.0542	0.0943
HANGSENG	24.11.1969	0.3447	0.1	0.4184	0.4584	0.3642	0.3204	0.1	0.3662	0.4078	0.3287	0.3483	0.1	0.4133	0.452	0.3714	0.1623	0.0584	0.0056	0.0032	0.0047
HEX	02.01.1995	0.4947	0.0529	0.0217	0.0145	0.0334	0.6187	0.0155	0.029	0.0206	0.047	0.4874	0.0491	0.0176	0.0115	0.0282	0	0	0	0	0
IBEX35	05.01.1987	0.5921	0.1	0.681	0.6983	0.7562	0.5908	0.1	0.5858	0.6057	0.7114	0.6113	0.1	0.7647	0.7878	0.8202	0.0001	0.0223	0.0003	0.0004	0.0013
ICEX	31.12.1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IPC	08.11.1991	0.7328	0.1	0.5202	0.4795	0.4845	0.7483	..1	0.6864	0.6387	0.5801	0.7366	0.1	0.5279	0.4876	0.4826	0	0	0	0	0
IPSA	02.01.1987	0.6088	0.1	0.2636	0.2564	0.3055	0.7016	0.1	0.4282	0.405	0.4445	0.6111	0.1	0.3019	0.2967	0.3494	0	0	0	0	0
JCI	04.04.1983	0.1847	0.1	0.0829	0.0659	0.104	0.2096	0.1	0.2209	0.197	0.253	0.1734	0.1	0.087	0.0697	0.1064	0	0	0	0	0
KLCI	03.01.1977	0.2759	0.1	0.2913	0.2733	0.3237	0.375	0.1	0.4667	0.468	0.5189	0.2867	0.1	0.3491	0.334	0.4003	0	0	0	0	0
KOSPI	04.01.1980	0.6223	0.1	0.5438	0.5356	0.5115	0.5535	0.1	0.5757	0.5869	0.5228	0.5791	0.1	0.3538	0.348	0.3667	0.0002	0	0.0001	0.0001	0
MDAX	29.02.1996	0.0505	0.0761	0.0858	0.1267	0.0483	0.0357	0.1	0.0579	0.0935	0.0329	0.0535	0.0763	0.0928	0.1365	0.0519	0.854	0.1	0.7616	0.7197	0.6502

Yearly rates of return – part 2

MERVAL	04.04.1988		0.1	0.1143	0.1184	0.0717	0	0.0001	0	0	0	0.0006	0.1	0.0235	0.0284	0.0128	0	0	0	0	0
MEXICIPC	20.12.1993	0.8358	0.1	0.683	0.6284	0.5925	0.7797	0.1	0.7445	0.6963	0.6765	0.8348	0.1	0.687	0.633	0.5824	0	0	0	0	0
MICEX	27.09.1997	0.0924	0.1	0.1559	0.1339	0.1009	0.6609	0.1	0.3521	0.3242	0.3665	0.1441	0.1	0.2237	0.1988	0.1542	0	0	0	0	0
NASDAQ 100	01.10.1985	0.513	0.0022	0.011	0.0083	0.0122	0.503	0.022	0.0071	0.0051	0.0085	0.506	0.0023	0.0088	0.0064	0.0099	0.0005	0	0	0	0
NASDAQCOMP	03.01.1988	0.0036	0.1	0.0138	0.0199	0.0062	0.0039	0.1	0.187	0.0256	0.008	0.0015	0.01	0.0122	0.0185	0.0046	0.0009	0.0101	0.0007	0.0004	0.0004
NIKKEI225	01.03.1914	0.1024	0.1	0.0515	0.0405	0.0588	0.2058	0.1	0.0621	0.0403	0.0859	0.1144	0.1	0.0353	0.0264	0.0457	0	0	0	0	0
NZX50	03.01.2001	0	0.0966	0.0234	0.037	0.0071	0.0001	0.1	0.0252	0.0388	0.0083	0	0.0919	0.0232	0.0365	0.0071	0	0	0	0	0
OMXRIGA	03.01.2000	0.0658	0.1	0.2997	0.3343	0.1337	0.0608	0.1	0.2358	0.2794	0.0947	0.0658	0.1	0.3013	0.3361	0.1347	0.0754	0.0001	0	0	0
OMXSTOCKHOLM	30.09.1986	0.2116	0.1	0.1538	0.2025	0.156	0.2298	0.0634	0.0651	0.0887	0.0844	0.1863	0.1	0.1484	0.2044	0.1464	0.002	0.0017	0.0005	0.0003	0.0006
OMXTALIN	03.01.2000	0.0011	0.1	0.0448	0.0718	0.0231	0.0011	0.1	0.434	0.693	0.0231	0.0013	0.1	0.0454	0.0728	0.0239	0.222	0.0055	0.0037	0.0032	0.0079
OMXVILNIUS	01.01.2000	0	0.0866	0.0115	0.007	0.0079	0	0.0333	0.0219	0.0145	0.0125	0	0.0249	0.0104	0.0063	0.0074	0	0	0	0	0
OSE	03.01.1983	0.06111	0.1	0.5837	0.6847	0.3559	0.0446	0.1	0.5075	0.629	0.3254	0.0618	0.1	0.5919	0.6903	0.3585	0	0	0	0	0
PSEI	02.01.1986	0.7539	0.1	0.2912	0.2619	0.2948	0.9439	0.1	0.5634	0.5196	0.5678	0.7316	0.1	0.3017	0.2723	0.3015	0	0	0	0	0
PSI20	31.12.1992	0.3487	0.21	0.3627	0.3869	0.3359	0.443	0.1	0.4444	0.4762	0.4716	0.3576	0.1	0.4514	0.4845	0.4135	0.5476	0.0283	0.051	0.0428	0.0675
PX50	07.09.1993	0.1091	0.1	0.6684	0.7452	0.5065	0.2932	0.1	0.7693	0.825	0.6173	0.0836	0.1	0.6338	0.7361	0.4926	0.9651	0.1	0.3207	0.2868	0.4311
RTS	01.09.1995	0.0152	0.1	0.0449	0.0717	0.0313	0.0212	0.1	0.0796	0.1314	0.046	0.0126	0.1	0.0448	0.03	0.03	0.0186	0.001	0.0006	0.0007	0.0005
RUSSEL	22.10.2001	0.4265	0.1	0.3788	0.4332	0.3769	0.4536	0.1	0.4949	0.5462	0.4828	0.4505	0.1	0.4583	0.517	0.4589	0.0405	0.0101	0.0008	0.001	0.0004
SAX	03.07.1995	0.04351	0.1	0.6193	0.602	0.4902	0.4391	0.1	0.508	0.5015	0.4491	0.4538	0.1	0.5842	0.5716	0.5043	0.0442	0	0.0001	0.0001	0.0001
SDAX	15.03.1999	0.2871	0.0862	0.0779	0.0969	0.0887	0.2926	0.0616	0.0699	0.0861	0.0836	0.2855	0.0766	0.0749	0.093	0.0877	0.5323	0.1	0.4582	0.4342	0.3701
SENSEX	03.04.1979	0.0626	0.1	0.9053	0.9346	0.7964	0.4826	0.1	0.8978	0.9138	0.831	0.4691	-.1	0.9684	0.9762	0.8859	0	0	0	0	0
SESESLCT	02.01.2003	0.7815	0.1	-.7622	0.7225	0.7293	0.7536	0.1	0.949	0.9411	0.9108	0.7836	0.1	0.7711	0.7334	0.749		0.0013	0	0	0
SET	02.07.1987	0.09529	0.1	0.5059	0.4725	0.5531	0.9136	0.1	0.4802	0.4527	0.5086	0.9504	0.1	0.5059	0.4737	0.5351	0.3412	0.0249	0.0006	0.0003	0.0007
SMI	01.07.1988	0.05614	0.1	0.2456	0.2703	0.2974	0.5876	0.1	0.1554	0.1667	0.188	0.6257	0.1	0.2568	0.2743	0.3317	0.0164	0.0003	0.0008	0.001	0.0009
SOFIX	26.11.2001	0	0.0136	0.0037	0.0038	0.0021	0	0.0183	0.0058	0.0064	0.003	0	0.0132	0.0035	0.0035	0.002	0.8339	0.0003	0.0005	0.0002	0.0017
SP500	02.01.1900	0	0.0707	0.0104	0.0291	0.0048	0	0.0322	0.009	0.0246	0.0042	0	0.0736	0.0114	0.0323	0.0048	0	0	0	0	0
SPTSXCOMP	03.01.1961	0.0121	0.1	0.02061	0.3356	0.1396	0.0872	0.1	0.6121	0.7262	0.4567	0.0393	0.1	0.4637	0.5996	0.2751	0	0	0	0	0
SSEBSHARE	04.01.2000	0.9708	0.1	0.5502	0.5071	0.5143	0.8614	0.1	0.8467	0.8272	0.7696	0.9691	0.1	0.5573	0.5121	0.5151	0.3781	0.0198	0.0318	0.013	0.0178
SSECOMP	19.12.1990	0.6804	0.0624	0.0768	0.0689	0.0696	0.9178	0.1	0.1377	0.1164	0.1222	0.6857	0.0585	0.0634	0.0492	0.0599	0.0001	0.0217	0.0009	0.0005	0.0008
STRAITSTIMES	28.12.1987	0.8503	0.1	0.9707	0.9663	0.9272	0.8552	0.1	0.9932	0.9931	0.9749	0.8734	0.1	0.9819	0.9792	0.9481	0.3392	0.0651	0.0387	0.0454	0.0488
TAIEX	05.01.1995	0.4281	0.0042	0.0092	0.0075	0.0144	0.4636	0.0075	0.0486	0.0493	0.0674	0.3551	0.006	0.0165	0.0153	0.0227	0.7351	0.0886	0.0174	0.0123	0.0285
TECDAX	16.09.1999	0.1056	0.014	0.0041	0.0054	0.0041	0.2498	0.0446	0.0085	0.0095	0.0103	0.1016	0.0141	0.0039	0.0051	0.004	0.5781	0.0214	0.053	0.0435	0.0731
TOPIX	22.10.2001	0.4627	0.1	0.3271	0.3264	0.3583	0.7187	0.1	0.4759	0.4683	0.5121	0.448	0.1	0.2819	0.2817	0.3094	0.7078	0.1	0.2528	0.242	0.3326
UX	03.11.1997	0.3761	0.1	0.494	0.5381	0.5114	0.5452	0.1	0.2958	0.3292	0.264	0.459	0.1	0.5141	0.5442	0.5395	0	0	0	0	0
WIG	16.04.1991	0	0.0001	0.0001	0.0001	0	0	0.0001	0.0001	0	0	0	0.0001	0.0001	0.0001	0	0.1013	0.0067	0.0016	0.0019	0.0021
XU100	02.01.1990	0.223	0.1	0.1443	0.1841	0.1124	0.2477	0.0958	0.1374	0.1728	0.1074	0.2558	0.1	0.2158	0.276	0.1726	0.0001	0.0028	0	0	0

Source: own calculations

Conclusion

- The obtained result permit to formulate the following conclusion:

- The higher the data compression (daily->weekly->monthly->quarterly->yearly), the less number of H_0 hypothesis rejection.

Verification of the hypothesis of normal distribution of returns for the following indexes: CAC40, DAX, DJIA, FTSE250, Nikkei and S&P500 when the investment horizon is equal to one year and during 28 up and down waves for DJIA index

Table 4. Results of testing the null hypothesis for each year (in the period of (2013-2016)

	Year	C-C					O-O					O-C					Overnight				
		J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D
DJIA	2013	0	**0.1**	**0.0626**	**0.0612**	0.0364	0	**0.1**	**0.0796**	**0.0763**	**0.533**	0	**0.1**	**0.053**	**0.501**	0.0299	0	0	0	0	0
	2014	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2015	0	**0.0697**	0.0159	0.0107	0.0205	0	**0.071**	0.0128	0.0082	0.0149	0	**0.1**	0.0139	0.0094	0.0158	0	0	0	0	0
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0.1**	0.0139	0.0108	0.0117
SP500	2013	0	0.0419	0.017	0.019	0.005	0	0.0083	0.005	0.0045	0.0019	0	**0.0918**	0.0075	0.0079	0.0023	0.0005	0	0	0	0
	2014	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0.1**	0.0091	0.0062	0.0048
	2015	0	0.0073	0.0007	0.0003	0.0008	0	0.0085	0.0011	0.0005	0.0013	0.0082	0.0012	0.0007	0.0011	0	0	0.03	0.0206	0.0135	0.0275
	2016	0	0	0	0	0	0	0	0	0	0	0	0.0001	0	0	0	0.0046	0.0385	**0.1472**	**0.1323**	**0.1836**
DAX	2013	0.0004	0.0075	0.004	0.0025	0.0009	0.0038	0.0042	0.0005	0.0004	0.0002	0	0.0017	0.0001	0.0001	0	0	0	0.0001	0	0
	2014	0.0351	0.0009	0.0001	0	0.0001	**0.1517**	0.0209	0.0137	0.0126	0.0123	0.01945	0.0015	0.0035	0.0031	0.0042	0	0.0146	0.0001	0.0001	0
	2015	**0.2053**	**0.1**	**0.0707**	**0.062**	**0.1028**	0.0349	0.0391	**0.0504**	**0.0562**	0.0355	0.0265	0.048	**0.0946**	0.0132	**0.1034**	0	0.0051	0.0003	0.0001	0.0001
	2016	0	0.005	0.0007	0.0006	0.0005	0	0	0	0	0	0	0.0051	0.0009	0.0004	0.0005	0	0	0	0	0
CAC40	2013	0.0001	**0.0858**	0.0074	0.0045	0.0058	0	**0.1**	0.0183	0.0209	0.0103	0	**0.1**	0.014	0.011	0.0084	0	0	0	0	0
	2014	0.0002	0.0146	0.0032	0.002	0.0023	0.0002	0.0008	0.0005	0.0003	0.0003	0	0.0034	0.0028	0.0024	0.0038	0	0.004	0.0001	0	0
	2015	0.0005	0.0326	0.0028	0.0017	0.002	0	0.00435	0.0062	0.0072	0.0023	0.0146	0.0409	0.0206	0.0303	0.0272	0	0	0	0	0
	2016	0	0.0004	0	0	0	0	0	0	0	0	0	**0.1**	**0.1406**	**0.1166**	**0.0894**	0	0	0	0	0
FTSE250	2013	0	**0.0768**	0.0125	0.0155	0.0079	0	**0.1**	0.0183	0.0209	0.0103	0	**0.1**	0.014	0.011	0.0084	0	0	0	0	0
	2014	**0.4816**	**0.1**	**0.6962**	**0.708**	**0.6796**	**0.4888**	**0.1**	**0.6872**	**0.6928**	**0.6745**	**0.4836**	**0.1**	**0.7217**	**0.7342**	**0.6991**	0	0	0	0	0
	2015	0	**0.1**	0.0145	0.0096	0.0106	0	**0.1**	0.0138	0.0092	0.0102	**0.1**	0.0145	0.0096	0.0096	0.0106	0	0	0	0	0
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Nikkei225	2013	0	**0.0786**	**0.0518**	**0.0887**	0.0153	0.0068	**0.1**	**0.3419**	**0.3491**	**0.2771**	0	0.0002	0.0001	0.00001	0	**0.0632**	**0.1**	**0.1423**	**0.1439**	**0.0576**
	2014	0.0005	0.0013	0.0001	0	0	0	**0.1**	**0.3697**	**0.3372**	**0.3716**	0	0.0031	0.0006	0.0003	0.0002	**0.2222**	0.0389	**0.0665**	**0.0918**	**0.0558**
	2015	0	0	0	0	0	0	0.002	0.0006	0.0007	0.0001	0	0.0001	0	0	0	**0.2228**	**0.1**	**0.2227**	**0.2803**	**0.0633**
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0.4999**	**0.0803**	**0.1679**	**0.1433**	**0.2268**

Source: own calculations

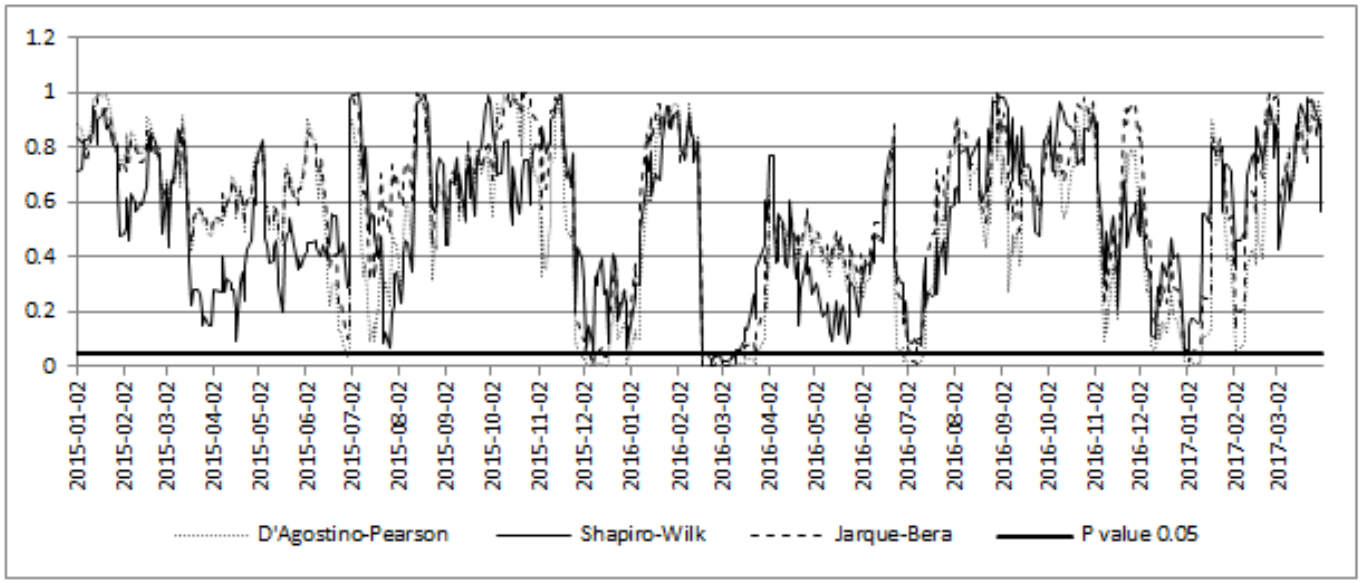
- If for individual index, at least two out of six tests do not allow to reject the null hypothesis, the distribution of returns represents a normal distribution in period of the analyzed years. Such outcomes were registered for:
  - DJIA: O-C (2013), O-O (2013) and O-C (2013),
  - DAX: C-C (2015), O-O (2015) and O-C (2015),
  - S&P 500: Overnight (2016),
  - FTSE250: C-C (2014), O-O (2014) and O-C (2014),
  - CAC40: O-C (2016),
  - NIKKEI225: C-C (2013), O-O (2013 and 2014) and Overnight (2013, 2014, 2015 and 2016).

Up and down waves of DJIA

	The wave begining	The wave end	Direction of price movement	C-C					O-O					O-C					Overnight				
				J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D
1	18.06.1901	15.10.1903	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	15.10.1903	18.01.1906	Up	0	0.0001	0	0	0	0	0.0001	0	0	0	0	0.0001	0	0	0	0	0.0001	0	0	0
3	18.01.1906	21.11.1907	Down	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	21.11.1907	05.11.1909	Up	0	0.0146	0.0004	0.0002	0.0001	0	0.0146	0.0004	0.0002	0.0001	0	0	0	0	0	0	0.0146	0.0004	0.0002	0.0001
5	05.11.1909	24.12.1914	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	24.12.1914	03.11.1919	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	03.11.1919	25.08.1921	Down	0	0.0366	0.0054	0.0044	0.0014	0	0.0366	0.0054	0.0044	0.0014	0	0	0	0	0	0	0.0366	0.0054	0.0044	0.0014
8	25.08.1921	29.08.1929	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	29.08.1929	09.07.1932	Down	0	0.0001	0	0	0	0	0.0001	0	0	0	0	0	0	0	0	0	0.0001	0	0	0
10	09.07.1932	10.03.1937	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	10.03.1937	29.04.1942	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	29.04.1942	14.12.1961	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	14.12.1961	26.06.1962	Down	0	0.0001	0	0	0	0	0.0004	0	0	0	0	0.0003	0	0	0	0	0	0	0	0
14	26.06.1962	11.02.1966	Up	0	0	0	0	0	0	0	0	0	0	0	0.0003	0	0	0	0	0	0	0	0
15	11.02.1966	11.10.1966	Down	0.8069	0.1	0.9254	0.9126	0.9216	0.8024	0.1	0.9568	0.9493	0.9551	0.7843	0.1	0.9115	0.8974	0.926	0	0	0	0	0
16	11.10.1966	06.12.1968	Up	0	0.1	0.4066	0.4751	0.1408	0.0016	0.1	0.6493	0.7777	0.3418	0.0017	0.1	0.7348	0.8286	0.4437	0	0	0	0	0
17	06.12.1968	26.05.1970	Down	0	0.0697	0.0495	0.0379	0.017	0.0012	0.0857	0.0948	0.0845	0.03	0.0017	0.1	0.1143	0.101	0.0464	0	0	0	0	0
18	26.05.1970	12.01.1973	Up	0	0.0002	0	0	0	0	0.0006	0	0	0	0	0.0011	0.0001	0	0	0	0	0	0	0
19	12.01.1973	10.12.1974	Down	0.0003	0.0449	0.068	0.1309	0.0301	0.0044	0.1	0.4198	0.6594	0.3007	0.0022	0.0513	0.1715	0.365	0.0845	0	0	0	0	0
20	10.12.1974	24.09.1976	Up	0.164	0.1	0.3175	0.4074	0.3102	0.1848	0.1	0.2754	0.3767	0.2114	0.0936	0.1	0.1432	0.1576	0.1202	0	0	0	0	0
21	24.09.1976	02.03.1978	Down	0.4173	0.0826	0.1705	0.1607	0.2233	0.8806	0.1	0.1582	0.1402	0.1797	0.4098	0.0923	0.1062	0.089	0.1247	0	0	0	0	0
22	02.03.1978	25.08.1987	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	25.08.1987	20.10.1987	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	20.10.1987	20.01.2000	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	20.01.2000	11.10.2002	Down	0	0.0083	0.0002	0.0001	0	0	0.0175	0.0002	0.0001	0	0	0.0034	0.0001	0	0	0	0	0	0	0
26	11.10.2002	12.10.2007	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	12.10.2007	10.03.2009	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	10.03.2009	31.03.2017	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Results- Part III

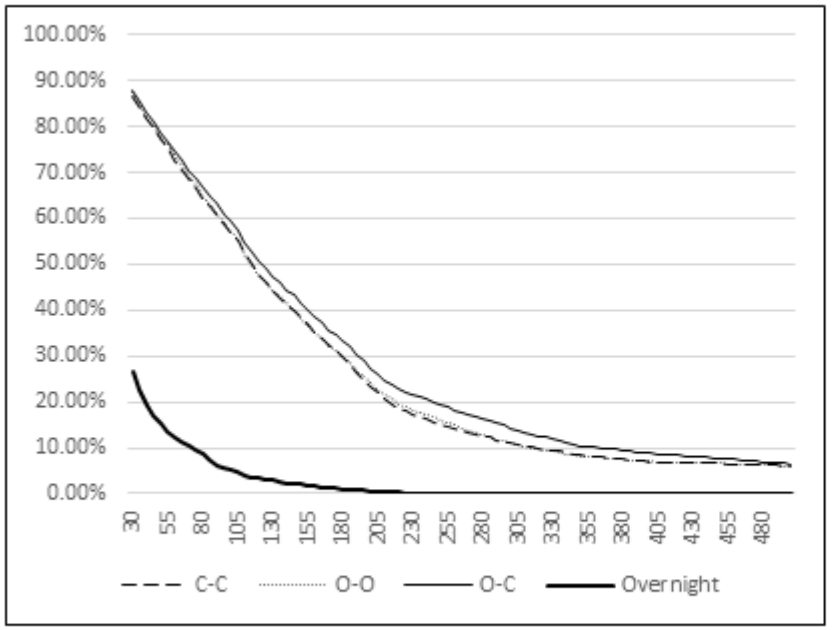
Figure 2. Value of parameter *p* for DJIA in the period 02.01.2015 - 31.03.2017 when carrying out three different tests and K = 30



Source: own calculations

Results Part III (DJIA)

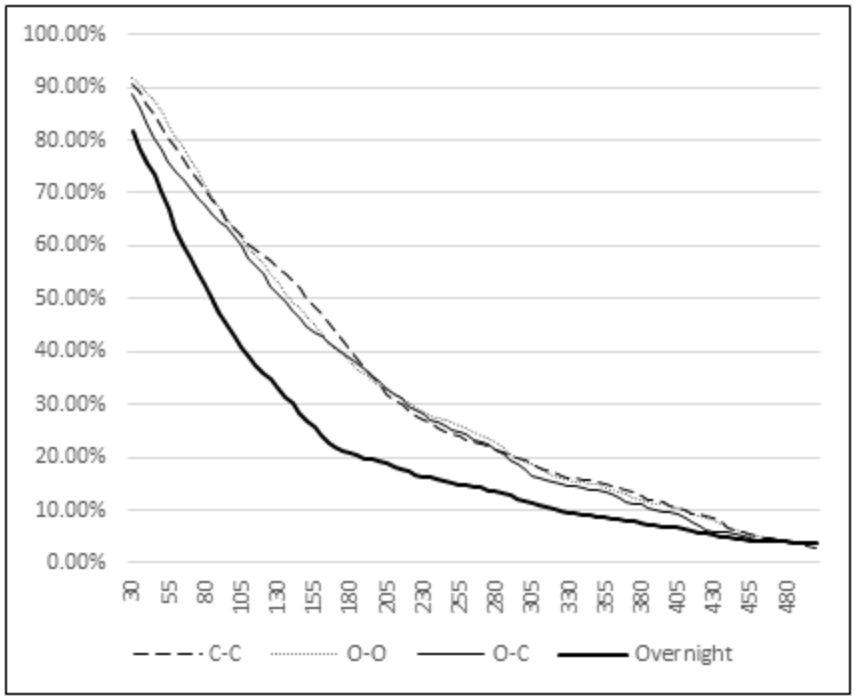
Figure 3. The percentage of cases where there was no reason for rejecting the null hypothesis for DJIA returns with the use of Jarque-Bera test, depending on K (change of K: every 1 unit)



Source: own calculations

Results – Part III (DAX)

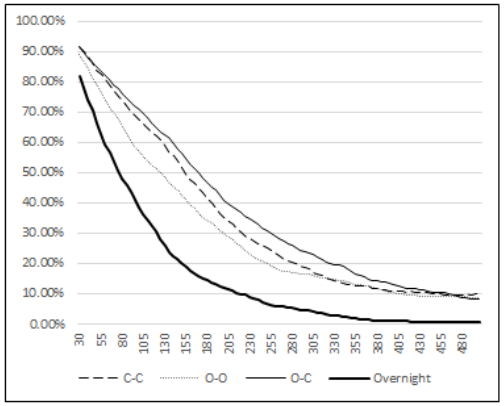
Figure 4. The percentage of cases where there was no basis for rejecting the null hypothesis for DAX returns with the use of Jarque-Bera test, depending on *K* (change of K: every 5 units)



Source: own calculations

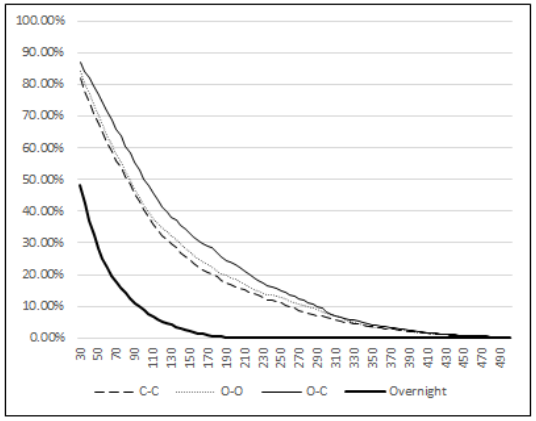
Results Part III

Figure 5. The percentage of cases where there was no basis for rejecting the null hypothesis for CAC40 index returns with the use of Jarque-Bera test, depending on *K* (change *K*: every 5 units)



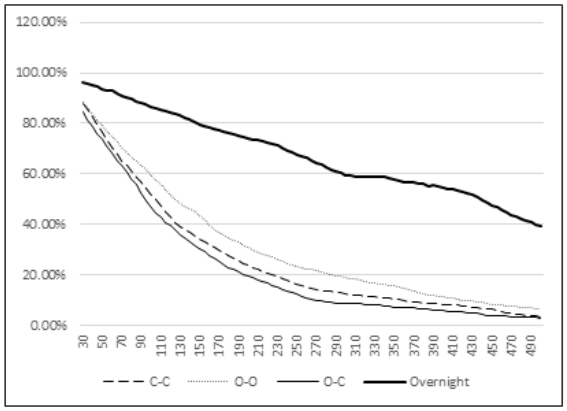
Source: own calculations

Figure 6. The percentage of cases where there was no basis for rejecting the null hypothesis for FTSE250 index returns with the use of Jarque-Bera test, depending on *K* (change *K*: every 5 units)



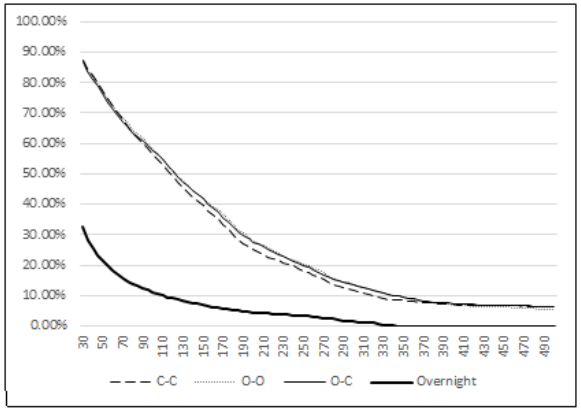
Source: own calculations

Figure 7. The percentage of cases where there was no basis for rejecting the null hypothesis for Nikkei index returns with the use of Jarque-Bera test, depending on *K* (change *K*: every 5 units)



Source: own calculations

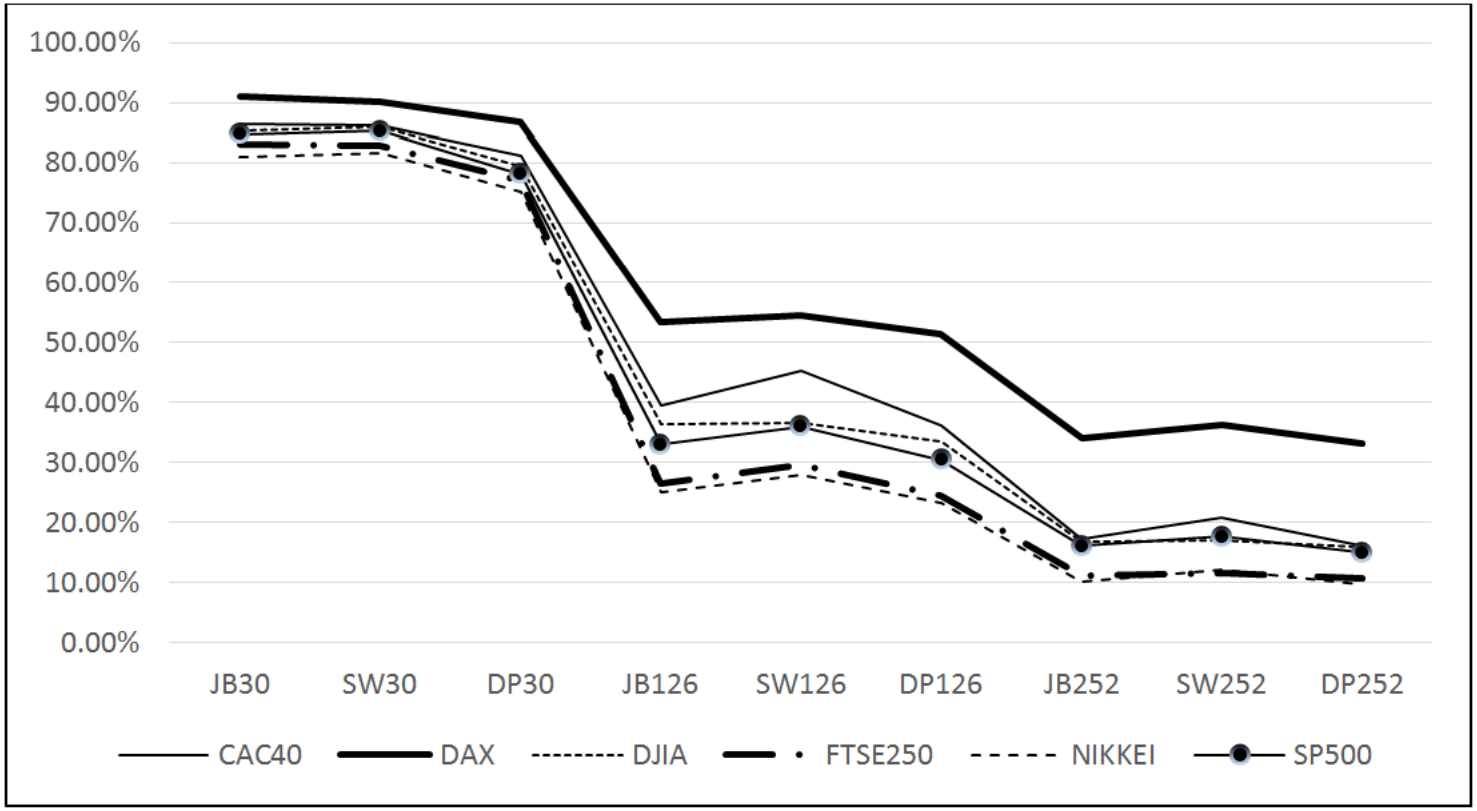
Figure 8. The percentage of cases where there was no basis for rejecting the null hypothesis for S&P 500 index returns with the use of Jarque-Bera test, depending on *K* (change *K*: every 5 units)



Source: own calculations

Results - Part III

Figure 9. The percentage of cases where there was no reason for rejecting the null hypothesis for six main indexes returns with the use of Jarque-Bera, D'Agostino-Pearson and Shapiro-Wilk tests, depending on *K* (K=30, 126 and 252 sessions) and C-C rates of return



Source: own calculations

Results – Part III

- For small *K*, the highest percentage of non-rejecting null hypothesis was observed for the DAX index, followed by CAC40, DJIA, S&P500, FTSE250 and NIKKEI – see figure 9.
- With the increase of the parameter *K*, this order remained stable. For K=252 the deference in percentage of non-rejection null hypothesis between DAX and NIKKEI was higher than for K=30.

Ranking of equity indices due to the proximity of their rates of return to the normal distribution

		30 sessions				126 sessions				252 sessions				
		C-C	O-O	O-C	OV	C-C	O-O	O-C	OV	C-C	O-O	O-C	OV	Total
1	AEX	8	24	9	6	4	15	5	6	3	10	3	5	1
2	All Ordinaries	15	14	35	49	11	7	20	38	4	4	11	23	13
3	Athex Composite	53	52	46	23	54	50	44	18	53	50	42	19	46
4	BEL20	24	30	36	9	22	34	43	10	27	37	39	11	25
5	BET	55	56	59	38	55	55	57	39	55	54	55	43	55
6	Bovespa	10	5	4	62	9	5	4	60	6	5	5	57	15
7	BUX	49	42	15	26	44	40	22	30	41	39	21	33	35
8	CAC40	29	36	7	7	29	36	7	8	26	32	6	9	15
9	CDAX	6	11	24	2	5	17	31	2	12	29	39	2	8
10	DAX	7	2	27	3	7	4	24	3	7	6	29	3	5
11	DJCA	25	27	44	32	20	25	39	28	16	23	37	29	29
12	DJIA	34	21	33	60	35	20	29	54	37	19	26	45	38
13	DJTA	43	43	48	50	43	39	46	42	39	34	40	37	46
14	DJUA	48	35	50	57	47	37	47	57	45	38	47	50	52
15	EOE	4	6	17	11	3	6	16	9	6	8	14	8	2
16	FSE100	50	35	31	47	50	42	36	51	50	45	43	53	50
17	FTSE250	28	8	35	13	30	9	30	12	35	12	27	13	18
18	FTSEMIBTEL	19	23	18	47	17	23	20	50	16	25	18	54	27
19	HANG SENG	23	19	26	5	18	19	27	5	17	17	19	6	11
20	HEX	58	57	61	51	58	57	62	55	60	59	62	60	61
21	IBEX	14	18	25	63	19	22	33	61	21	19	36	61	32
22	ICEX	37	46	37	55	37	42	34	52	32	40	31	47	44
23	IPC	60	61	53	40	59	61	52	43	57	58	52	46	57
24	IPSA	52	55	52	33	54	56	54	36	54	56	56	36	54
25	JCI	48	37	12	10	41	32	9	7	31	24	9	7	22
26	KLCI	9	10	12	12	13	16	15	14	19	22	20	21	9
27	KOSPI	39	31	47	54	42	38	48	46	43	43	50	41	49
28	MDAX	30	28	43	56	31	27	45	50	33	30	46	40	42
29	MERVAL	51	51	49	25	51	52	50	24	48	51	48	22	49
30	MEXICIPC	19	17	28	39	15	14	18	37	14	13	13	32	21
31	MICEX	54	53	56	4	52	52	55	4	52	49	54	4	43
32	NASDAQ100	16	9	9	59	12	8	8	59	8	7	7	55	20
33	NASDAQCOMP	59	60	58	61	60	58	58	62	60	58	57	62	62
34	NIKKEI	14	15	19	29	16	13	18	23	13	9	12	16	10
35	NZX50	56	54	55	43	56	54	56	45	56	55	58	50	56
36	OMXRIGA	64	64	62	48	64	64	61	57	64	63	59	58	63
37	OMXSTOCKOLM	21	12	1	37	23	19	12	32	28	20	24	26	19
38	OMXTALIN	46	47	45	42	50	47	49	42	51	52	51	42	53
39	OMXVILNUS	36	48	42	19	33	46	37	21	30	42	26	26	37
40	OSE	26	26	29	19	33	29	32	15	34	31	30	15	24
41	PSEI	44	44	54	37	45	44	53	42	47	45	50	51	51
42	PSI20	31	45	40	17	37	48	40	17	40	49	42	19	39
43	PX50	41	25	38	34	40	26	38	32	42	27	34	29	37
44	RTS	65	65	65	53	65	65	65	59	65	65	65	59	65
45	RUSSEL	42	51	51	15	48	53	51	20	49	53	53	27	47
46	SAX	33	23	10	27	28	21	10	22	23	11	9	21	16
47	SDAX	45	38	16	30	46	34	14	35	46	35	18	39	33
48	SEECOM	17	29	20	24	24	29	23	29	36	34	32	38	28
49	SENSEX	57	58	57	41	57	59	59	48	58	61	61	56	58
50	SESESLCT	63	63	64	58	63	64	64	63	63	64	64	63	65
51	SET	61	59	60	45	61	61	60	48	61	60	60	52	59
52	SMI	27	49	13	31	26	49	13	27	24	49	15	24	30
53	SOFIX	35	33	41	28	38	44	42	33	44	41	45	34	41
54	SP500	5	20	14	14	7	24	11	26	9	28	18	35	12
55	SPTSXCOM	40	42	2	16	34	31	3	13	23	21	4	10	17
56	SSEBSHARE	62	62	63	45	62	62	63	44	62	63	63	45	60
57	Straits Times	38	40	40	20	39	45	42	19	38	46	44	19	40
58	TAIEX	3	14	5	8	8	11	6	11	11	15	10	14	3
59	TECDAX	2	1	3	52	2	1	2	53	2	3	2	50	7
60	TOPIX	21	33	21	22	22	35	27	25	26	37	28	30	26
61	TSE300	12	3	32	1	14	2	28	1	19	2	33	1	6
62	UK100	11	16	30	64	11	13	27	64	10	14	24	64	31
63	UX	1	4	6	35	1	3	1	34	1	1	1	31	4
64	WIG	22	7	23	65	25	10	35	65	29	16	35	65	34
65	XU100	33	40	23	21	27	31	22	16	21	26	22	13	23

For example, for K = 30 sessions and C-C rates of return the first three places were ranked as follows: UX, TECDAX and TAIEX, while the last three were listed in the order: SESESLCT, OMXRIGA and RTS.

In the total ranking, the top three places were: AEX, EOE and TAIEX, and the last three: OMXRIGA and equally placed: RTS and SESESLCT

Conclusions

- Some of the conducted calculations prove unequivocally that the distribution of daily returns of equity indexes is not a normal distribution, thus confirming the results obtained by other researchers such as Kendall (1953), Fama (1976), Barunika et al. (Barunik, Vacha, Vosvrda, 2010).
- This remark applies to C-C rates of return

Conclusions

- The research shows also that the distribution of the remaining daily returns, e.g. O-O, C-O and overnight, calculated for the analyzed equity indexes does not follow a path of a normal distribution.

Conclusions

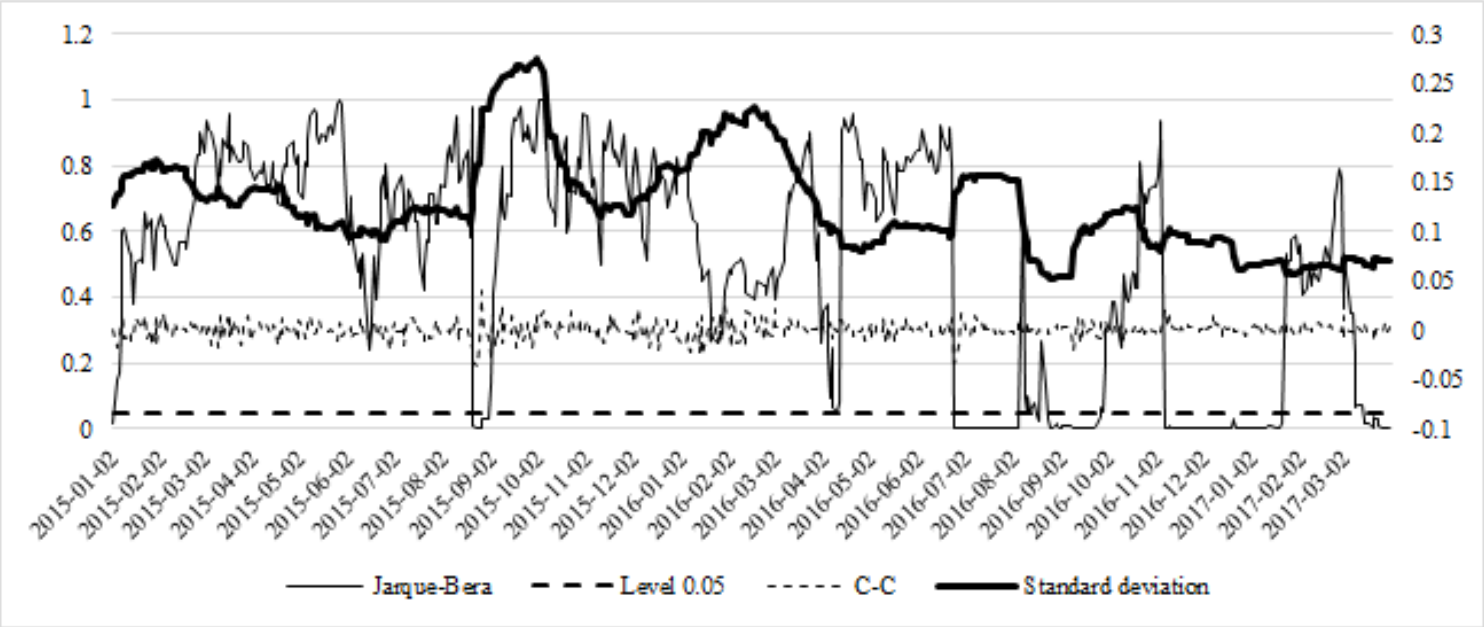
- It has been proved that the distribution of returns can be normal only in given time intervals.
- Time intervals can be set as individual years or up and down waves.
- The obtained results are consistent with those of Piasecki and Tomasik [Piasecki, Tomasik 2013] who proved the normal distribution of returns in certain upward and downward price movements on the Polish market.

Conslusions – stock index ranking

- As a result, it was found that the position of the index in the ranking is not dependent on the date of its first publication, and hence on the number of rates of return possible to calculate for analyzed index, but on the distribution of these rates of return.

- Analysis of the results obtained for K = 30 sessions concludes that for such short time interval, a sharp index change leads to a violent decrease in the value of parameter *p*.

Figure 8. The p-parameter chart for the DJIA index, K = 30, the Jarque-Bera test, the return rate (C-C) and the annualized standard deviation (p-value and level 0.05- left scale, C-C return and standard deviation - right scale)



Source: own calculations

- For example, with a strong increase in volatility on 19.08.2015, the value of *p* dropped below the trigger value of 0.05. Explaining the decrease in the value of parameter *p* below 0.05 for K = 126 and K = 252 sessions becomes more complex issue and requires further investigation.

Further research

- Other markets
  - commodities
  - FX

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