Extracting the Information Shocks form the Bank of England Inflation Density Forecasts

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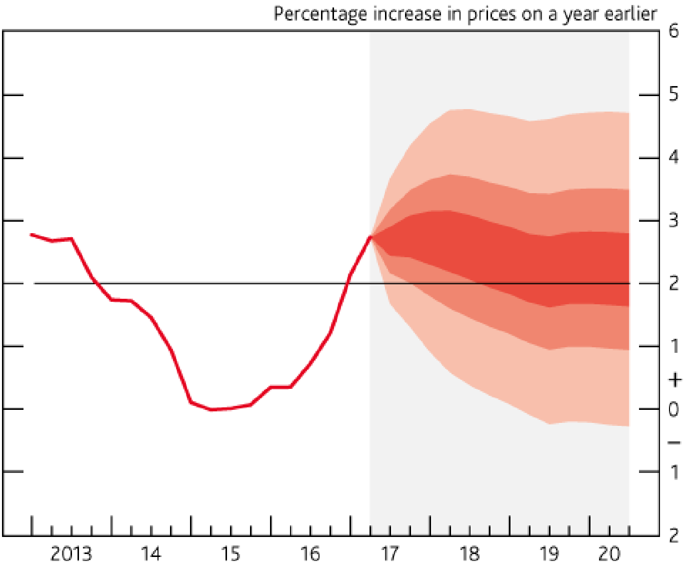
Motivation: Bank of England's Inflation Fan Chart



Motivation: Bank of England's Inflation Fan Chart

- The quarterly forecast round lasts for around six weeks.
- Structured as a series of discussions between the Monetary Policy Committee (MPC) and other staff members.
- Milestones:
  - **Publication of the Quarterly National Accounts** data by the ONS.
  - **The Benchmark meeting** (3 weeks before forecast publication):
    - First formal forecasts presented to the MPC.
    - Review of new data released since last forecast and latest international forecasts.
  - **The Key Issues meeting** (2 weeks before forecast publication):
    - Detailed assessment of core aspects of the forecasts.
    - Revision of assumptions, etc.
  - **The Draft meeting** (1 week before forecast publication):
    - Staff will revise forecasts.
    - New forecasts presented to the MPC.
  - **Sign-off of the final forecasts** as the 'best collective judgement' of the MPC.

Motivation: Bank of England's Inflation Fan Chart

- Density forecasts are constructed using the Two Piece Normal (TPN) distribution.

$$f(x) = \begin{cases} A \exp\left[-\frac{(x-\mu)^2}{2\sigma_1^2}\right] & \text{if } x < \mu \\ A \exp\left[-\frac{(x-\mu)^2}{2\sigma_2^2}\right] & \text{otherwise} \end{cases} \tag{1}$$

  with $A = \sqrt{2/\pi}(\sigma_1 + \sigma_2)^{-1}$.

- The parameters of the TPN for the following 12 quarters are released in the Quarterly Inflation Report ▸ QIR.
- Staff produces forecasts using a DSGE model (COMPASS since November 2011).
- There is a component of judgement in published forecasts.

Motivation: Bank of England's Inflation Fan Chart

- **Research problem**: How to extract the density of the 'shocks of information' perceived by the MPC between two releases.
- Why?
  - We can obtain a measure of (*ex ante*) inflation uncertainty.
  - We can test dynamically the efficiency of forecasts.
  - We can measure the incorporation of news to subsequent forecasts.
- **(Non-comprehensive) literature review:**
  - **Uncertainty measures:** Jurado, Ludvigson and Ng (2015); Bloom (2009); Baker, Bloom and Davis (2013); Clements (2014), Giordani and Soderling (2003); Lahiri and Liu (2006); Patton and Timmermann (2010); Charemza, Díaz and Makarova (2015).
  - **Evaluation of the BoE fan charts**: Clements (2004); Elder et al. (2005); Wallis (2004); Mitchell and Hall (2005); Dowd (2008); Mitchell and Wallis (2011); Gneiting and Ranjan (2011); Galbraith and van Norden (2012); Fawcet et al. (2015); Bank of England (2015).
  - **Forecast efficiency based on revisions:** Mankiw and Saphiro (1986); Nordhaus (1987); Mitchell (2008); Patton and Timmermann (2012).

Outline

1. Extracting the density of shocks via convolutions.
2. Measure of *ex-ante* inflation uncertainty.
3. Measure of information absorption.
4. Empirical results.
5. Conclusions and further work.

Extracting the Density of the Shocks of Information

- Assume we want to forecast the density of a random variable z_{t+h}.
- At moment t a forecast for z_{t+h}, say $z_{t+h|t}$, is produced in the form of a density forecast $f_{t+h|t}(\cdot)$.
- At moment $t+1$, a new forecast for z_{t+h}, say $z_{t+h|t+1}$, is also produced in the form of a density forecast $f_{t+h|t+1}(\cdot)$.
- $z_{t+h|t+1}$ is a revision of $z_{t+h|t}$ such that

$$z_{t+h|t+1} = z_{t+h|t} + \epsilon_{t+h|t+1}, \tag{2}$$

- $\epsilon_{t+h|t+1}$ can be interpreted as the shock of information perceived by the forecaster between t and $t+1$ and incorporated into the forecast.
- Can we recover the density of $\epsilon_{t+h|t+1}$?

Convolutions

- Notice that

$$\epsilon_{t+h|t+1} = z_{t+h|t+1} - z_{t+h|t}, \tag{3}$$

- Let X and Y be two independent random variables with densities $f_X(\cdot)$ and $f_Y(\cdot)$. The density of $Z = X - Y$ is

$$f_Z(t) = (f_X * f_Y)(t) = \int_{-\infty}^{\infty} f_X(\tau) f_Y(\tau - t) d\tau$$

- Let X and Y be two dependent random variables with joint density $f_{XY}(\cdot, \cdot)$. The density of $Z = X - Y$ is

$$f_Z(t) = \int_{-\infty}^{\infty} f_{XY}(\tau, \tau - t) d\tau$$

Copulas: Specification of the Joint Density

- Sklar (1959): Given the marginal cdfs $F_{t+h|t}(\cdot)$ and $F_{t+h|t+1}(\cdot)$, then

$$\tilde{F}_{z_{t+h|t+1}z_{t+h|t}}(x_1, x_2) = C\left[F_{t+h|t+1}(x_1), F_{t+h|t}(x_2)|\theta\right], \qquad x_1, x_2 \in \Re \tag{4}$$

  where the bivariate copula $C(\cdot, \cdot|\theta)$ is unique if the marginals are continuous and $\theta \in \Theta^m$.

- Joint probability density function

$$\tilde{f}_{z_{t+h|t+1}z_{t+h|t}}(x_1, x_2|\theta) = c\left[F_{t+h|t+1}(x_1), F_{t+h|t}(x_2)|\theta\right] f_{t+h|t+1}(x_1) f_{t+h|t}(x_2), \tag{5}$$

  where

$$c(u_1, u_2|\theta) = \frac{\partial^2 C(u_1, u_2|\theta)}{\partial u_1 \partial u_2}.$$

- The density of $\epsilon_{t+h|t+1}$ could be recovered as:

$$\tilde{f}_{\epsilon,t+1}(z|\theta) = \int_{-\infty}^{\infty} \tilde{f}_{z_{t+h|t+1}z_{t+h|t}}(x, x-z|\theta)dx. \tag{6}$$

Copulas: Estimation of the Copula Parameter

- The parameters θ in (5) can be estimated by maximum likelihood (IFM method, Joe and Xu, 1996) but
  - We don't (necessarily) have data.
  - We would need to assume parameter constancy.
- As $z_{t+h|t}$ and $\epsilon_{t+h|t+1}$ are independent by construction, we can approximate $f_{t+h|t+1}(\cdot)$ as

$$\tilde{g}_{t+h|t+1}(z|\theta) = \int_{-\infty}^{\infty} f_{t+h|t}(z-x) f_{\epsilon,t+1}(x|\theta) dx. \tag{7}$$

- We can estimate θ every revision minimizing the KLIC (Mitchell and Hall, 2005):

$$\hat{\theta}_{MDE} = \arg\min_{\theta} \int_{-\infty}^{\infty} f_{t+h|t+1}(x) \ln \left[\frac{f_{t+h|t+1}(x)}{\tilde{g}_{t+h|t+1}(x|\theta)} \right] dx. \tag{8}$$

- For every revision and every forecast horizon we have an estimate of θ.

Advantages of the MDE Estimator

- Only two consecutive density forecasts for the same fixed point event are needed.
- A different copula parameter is estimated for every vintage (and forecast horizon).
- Dependence between revisions is dynamic.
- The density of the information shock can be obtained for events that have not yet happened (*ex ante* measures).
- The method provide consistent estimators of the copula parameter (Akaike, 1973; Soofi, 2000).

A Measure of News Absorbtion

- By the convolution theorem (e.g. Bracewell, 1999), the Fourier transform of $f_{t+h|t+1}(\cdot)$ is

$$FT[f_{t+h|t+1}(z)] \equiv s_{t+h|t+1}(\lambda) = s_{t+h|t}(\lambda) s_{\epsilon,t+1}(\lambda), \qquad \lambda \in [-\pi, \pi]. \tag{9}$$

where

$$s_{t+h|t+1}(\lambda) = \int_{-\infty}^{\infty} f_{t+h|t+1}(z) e^{-i\lambda z} dz$$

- Measure of news absorbtion at frequency λ:

$$d_\epsilon(\lambda) = \frac{\ln[s_{\epsilon,t+1}(\lambda)]}{\ln[s_{t+h|t+1}(\lambda)]}, \qquad 0 \leq d_\epsilon(\lambda) \leq 1.$$

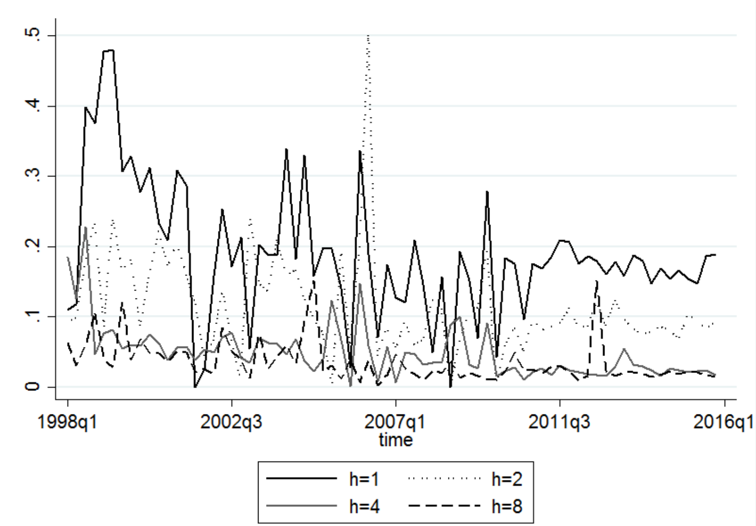
- Aggregating at all frequencies

$$Abs_{t+h|t+1} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d_\epsilon(\lambda) d\lambda \tag{10}$$

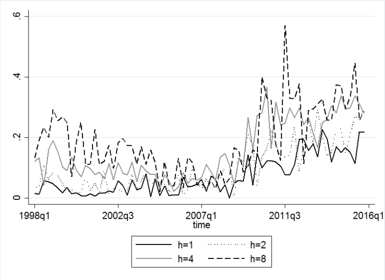
Application: Database Description

- The Bank of England produces inflation density forecasts since 1993.
- Forecasts produced for the next 12 quarter (8 until 2013Q2)
- Inflation targeting starts in 1992.
- BoE targeted RPIX inflation until December 2003 (CPI since).
- MPC granted independence in 1998.
- Period studied: 1998Q1-2015Q4.
- Results for constant interest rates assumption.
- Density of the information shocks extracted for every vintage and forecast horizon.
- Copula used: Frank (Frank, 1979; Genest, 1987).
  - Allows for negative dependence.
  - Symmetric dependence.
  - No tail dependence.

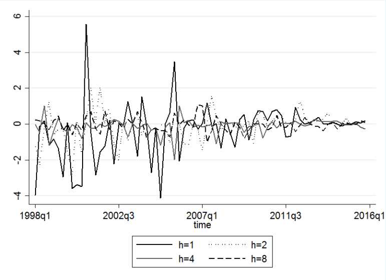
Estimation Results: KLIC Values at the Minimum



Moments of the Information Shocks



(a) variance

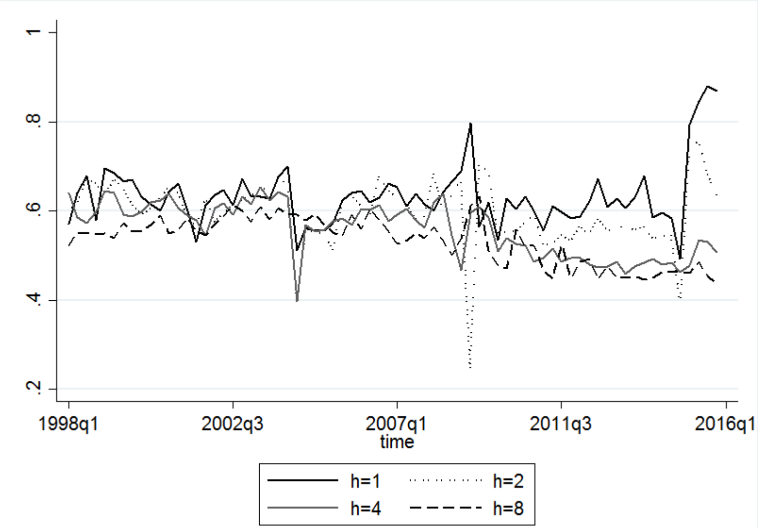


(b) skewness

Correlation of $Var[\epsilon_{t+h}(h)]$ with uncertainty measures

h	Correlation			p-value		
	EPU	JLN-F	JLN-M	EPU	JLN-F	JLN-M
1	0.6634	-0.2778	-0.3664	0.00	0.01	0.00
2	0.6764			0.00		
4	0.6162	0.0079	-0.3526	0.00	0.47	0.00
8	0.5215			0.00		

Measures of News Absorbtion



Robustness (Vintage: 2015Q4)

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
				Gaussian				
$\hat{\theta}$	0.999	0.999	0.999	0.999	0.999	0.990	0.999	0.999
KL divergence	0.3644	0.2479	0.1323	0.0550	0.0428	0.0225	0.0418	0.0618
Mean	-0.2338	-0.3611	-0.4770	-0.5786	-0.5916	-0.6172	-0.5852	-0.5416
Variance	0.0739	0.0891	0.0274	0.0126	0.0243	0.0389	0.0310	0.0177
Skewness	0.7307	1.3128	5.4910	3.0900	2.6214	-0.5947	1.8751	2.6683
				Frank				
$\hat{\theta}$	13.1095	21.2762	27.4918	21.5263	16.1350	18.6482	20.6416	17.1974
KL divergence	0.1891	0.0770	0.0288	0.0135	0.0118	0.0085	0.0105	0.0143
Mean	-0.2599	-0.3704	-0.4720	-0.5721	-0.5820	-0.6171	-0.5798	-0.5300
Variance	0.2176	0.1897	0.1575	0.2210	0.3465	0.2952	0.2493	0.3550
Skewness	0.1801	0.2636	0.3390	-0.0216	-0.0913	-0.1209	-0.1392	-0.0861
				Farlie-Gumbel-Morgenstern				
$\hat{\theta}$	1	1	1	1	1	1	1	1
KL divergence	0.3426	0.2593	0.2070	0.1658	0.1507	0.1471	0.1500	0.1590
Mean	-0.2600	-0.3703	-0.4713	-0.5729	-0.5861	-0.6283	-0.5917	-0.5476
Variance	0.8116	1.3960	1.9625	2.3160	2.4741	2.5512	2.6261	2.7686
Skewness	0.0561	0.0420	0.0319	-0.0036	-0.0268	-0.0592	-0.0792	-0.0739
				Clayton				
$\hat{\theta}$	38.1592	50.0000	34.8246	14.5060	32.2263	11.8634	17.1247	32.9567
KL divergence	0.3466	0.2062	0.1354	0.0881	0.0516	0.0626	0.0529	0.0666
Mean	-0.2213	-0.3263	-0.4715	-0.5822	-0.5877	-0.6350	-0.6070	-0.5438
Variance	0.1397	0.1727	0.1071	0.1497	0.0763	0.1788	0.1177	0.0578
Skewness	0.4184	1.1836	0.4278	-0.5534	-0.0029	-0.9820	-0.7467	0.9347
				Gumbel				
$\hat{\theta}$	9.2506	50.1587	13.5746	10.1426	9.4449	9.1797	10.0401	11.7277
KL divergence	0.3340	0.1808	0.1349	0.0636	0.0378	0.0255	0.0360	0.0530
Mean	-0.2890	-0.4679	-0.4888	-0.5833	-0.5851	-0.6265	-0.5888	-0.5318
Variance	0.1148	0.1263	0.0646	0.0682	0.0750	0.0801	0.0728	0.0611
Skewness	0.3956	0.6109	1.8140	0.9706	0.3930	0.3429	0.2493	0.6306

Conclusions

- A method to extract the density of the shocks of information is proposed.
- The method is based on density forecast revisions.
- A new measure of *ex-ante* inflation uncertainty is proposed.
- A measure of news absorbtion is proposed.
- Extensions:
  - Dynamic efficiency tests of density forecasts (Mincer-Zarnowitz, Nordhaus).
  - Effect of inflation uncertainty on macroeconomic and financial variables.
  - Uncertainty spillovers.